Chapter 11 Drill

The answers and explanations can be found in Chapter 17.

Section I: Multiple Choice

1. Which of the following is/are characteristics of simple harmonic motion?

    I. The acceleration is constant.
    II. The restoring force is proportional to the displacement.
    III. The frequency is independent of the amplitude.

    (A) II only
    (B) I and II only
    (C) I and III only
    (D) II and III only
    (E) I, II, and III

2. What combination of values will result in a spring–block system with the greatest frequency?

    (A) A = 50 cm; k = 80 N/m; m = 2 kg
    (B) A = 50 cm; k = 100 N/m; m = 2 kg
    (C) A = 50 cm; k = 100 N/m; m = 4 kg
    (D) A = 75 cm; k = 80 N/m; m = 2 kg
    (E) A = 75 cm; k = 100 N/m; m = 4 kg

3. A block attached to an ideal spring undergoes simple harmonic motion about its equilibrium position ($x = 0$) with amplitude A. What fraction of the total energy is in the form of kinetic energy when the block is at position $x = \frac{1}{2}A$?

    (A) $\frac{1}{3}$
    (B) $\frac{3}{8}$
    (C) $\frac{1}{2}$
    (D) $\frac{2}{3}$
    (E) $\frac{3}{4}$

4. A student measures the maximum speed of a block undergoing simple harmonic oscillations of amplitude A on the end of an ideal spring. If the block is replaced by one with twice the mass but the amplitude of its oscillations remains the same, then the maximum speed of the block will

    (A) decrease by a factor of 4
    (B) decrease by a factor of 2
    (C) decrease by a factor of $\sqrt{2}$
    (D) remain the same
    (E) increase by a factor of 2

5. A spring with a natural length of 40 cm and a spring constant of 400 N/m is hung vertically with a 10 kg mass attached to the end. Assuming the spring's mass is negligible, what will be the final length of the spring when it reaches equilibrium?

    (A) 25 cm
    (B) 35 cm
    (C) 40 cm
    (D) 50 cm
    (E) 65 cm

6. A linear spring of force constant k is used in a physics lab experiment. A block of mass m is attached to the spring and the resulting frequency, f, of the simple harmonic oscillations is measured. Blocks of various masses are used in different trials, and in each case, the corresponding frequency is measured and recorded. If f^2 is plotted versus $1/m$, the graph will be a straight line with slope

    (A) $4\pi^2/k^2$
    (B) $4\pi^2/k$
    (C) $4\pi 2k$
    (D) $k/4\pi^2$
    (E) $k^2/4\pi^2$

7. A block of mass $m = 4$ kg on a frictionless, horizontal table is attached to one end of a spring of force constant $k = 400$ N/m and undergoes simple harmonic oscillations about its equilibrium position ($x = 0$) with amplitude $A = 6$ cm. If the block is at $x = 6$ cm at time $t = 0$, then which of the following equations (with x in centimeters and t in seconds) gives the block's position as a function of time?

(A) $x = 6\sin(10t + \frac{1}{2}\pi)$
(B) $x = 6\sin(10\pi t + \frac{1}{2}\pi)$
(C) $x = 6\sin(10\pi t - \frac{1}{2}\pi)$
(D) $x = 6\sin(10t)$
(E) $x = 6\sin(10t - \frac{1}{2}\pi)$

8. A block attached to an ideal spring undergoes simple harmonic motion about its equilibrium position with amplitude A and angular frequency ω. What is the maximum magnitude of the block's velocity?

(A) $A\omega$
(B) $A^2\omega$
(C) $A\omega^2$
(D) A/ω
(E) A/ω^2

9. A simple pendulum swings about the vertical equilibrium position with a maximum angular displacement of 5° and period T. If the same pendulum is given a maximum angular displacement of 10°, then which of the following best gives the period of the oscillations?

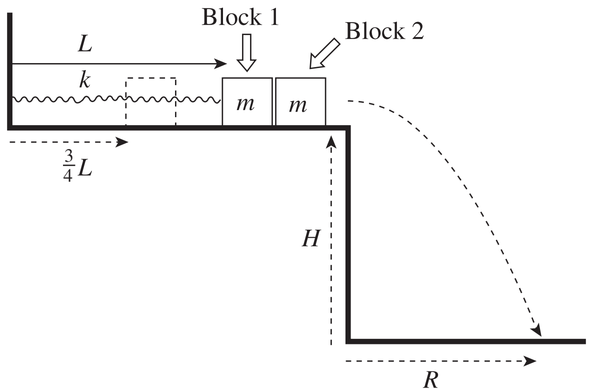
(A) $T/2$
(B) $T/\sqrt{2}$
(C) T
(D) $T\sqrt{2}$
(E) $2T$

10. Which of the following best describes the relationship between the tension force, F_T, in the string of a pendulum and the component of gravity that pulls antiparallel to the tension, F_G? Assume that the pendulum is only displaced by a small amount.

(A) $F_T > F_G$
(B) $F_T \geq F_G$
(C) $F_T = F_G$
(D) $F_T \leq F_G$
(E) $F_T < F_G$

Section II: Free Response

1. The figure below shows a block of mass m (Block 1) that's attached to one end of an ideal spring of force constant k and natural length L. The block is pushed so that it compresses the spring to 3/4 of its natural length and then released from rest. Just as the spring has extended to its natural length L, the attached block collides with another block (also of mass m) at rest on the edge of the frictionless table. When Block 1 collides with Block 2, half of its kinetic energy is lost to heat; the other half of Block 1's kinetic energy at impact is divided between Block 1 and Block 2. The collision sends Block 2 over the edge of the table, where it falls a vertical distance H, landing at a horizontal distance R from the edge.



(a) What is the acceleration of Block 1 at the moment it's released from rest from its initial position? Write your answer in terms of k, L, and m.

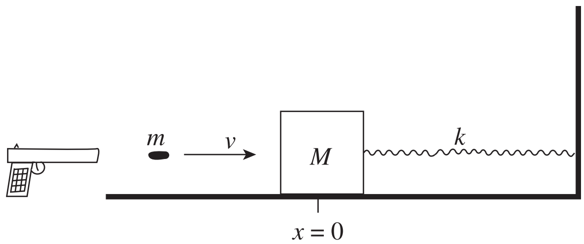
(b) If v_1 is the velocity of Block 1 just before impact, show that the velocity of Block 1 just after impact is $\frac{1}{2}v_1$.

(c) Determine the amplitude of the oscillations of Block 1 after Block 2 has left the table. Write your answer in terms of L only.

(d) Determine the period of the oscillations of Block 1 after the collision, writing your answer in terms of T_0, the period of the oscillations that Block 1 would have had if it did not collide with Block 2.

(e) Find an expression for R in terms of H, k, L, m, and g.

2. A bullet of mass m is fired horizontally with speed v into a block of mass M initially at rest, at the end of an ideal spring on a frictionless table. At the moment the bullet hits, the spring is at its natural length, L. The bullet becomes embedded in the block, and simple harmonic oscillations result.



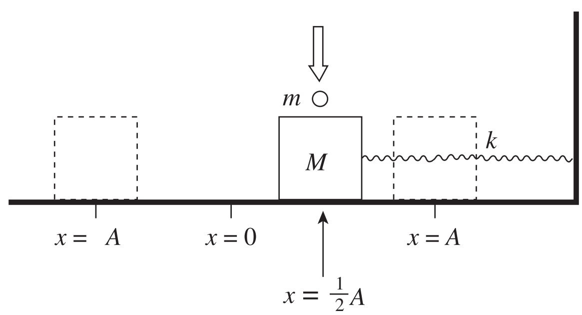
(a) Determine the speed of the block immediately after the impact by the bullet.

(b) Determine the amplitude of the resulting oscillations of the block.

(c) Compute the frequency of the resulting oscillations.

(d) Derive an equation which gives the position of the block as a function of time (relative to $x = 0$ at time $t = 0$).

3. A block of mass M oscillates with amplitude A on a frictionless horizontal table, connected to an ideal spring of force constant k. The period of its oscillations is T. At the moment when the block is at position $x = \frac{1}{2}A$ and moving to the right, a ball of clay of mass m dropped from above lands on the block.



(a) What is the velocity of the block just before the clay hits?

(b) What is the velocity of the block just after the clay hits?

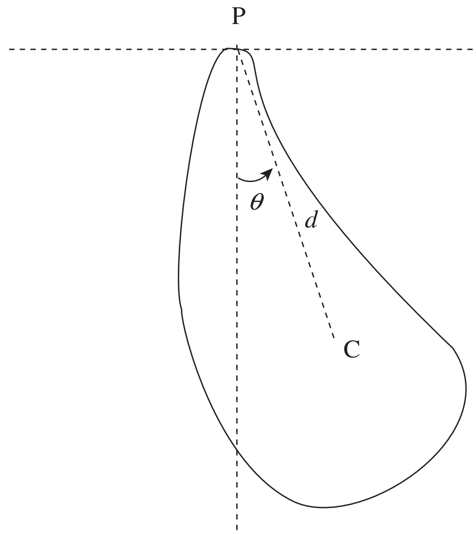
(c) What is the new period of the oscillations of the block?

(d) What is the new amplitude of the oscillations? Write your answer in terms of A, k, M, and m.

(e) Would the answer to part (c) be different if the clay had landed on the block when it was at a different position? Support your answer briefly.

(f) Would the answer to part (d) be different if the clay had landed on the block when it was at a different position? Support your answer briefly.

4. An object of total mass M is allowed to swing around a fixed suspension point P. The object's moment of inertia with respect to the rotation axis perpendicular to the page through P is denoted by I. The distance between P and the object's center of mass C, is d.



(a) Compute the torque τ produced by the weight of the object when the line PC makes an angle θ with the vertical. (Take the counterclockwise direction as positive for both θ and τ.)

(b) If θ is small, so that $\sin\theta$ may be replaced by θ, write the restoring torque τ computed in part (a) in the form $\tau = -\kappa\theta$.

A simple harmonic oscillator whose displacement from equilibrium, z, satisfies an equation of the form $\frac{d^2z}{dt^2} = -bz$ has a period of oscillation given by the formula $T = \frac{2\pi}{\sqrt{b}}$.

(c) Setting z equal to θ in the equation above, use the result of part (b) to derive an expression for the period of small oscillations of the object shown above.

(d) Answer the question posed in part (c) if the object were a uniform bar of mass M and length L (whose moment of inertia about one of its ends is given by the equation $I = \frac{1}{3}ML^2$.)