CALCULUS AB

SECTION I, Part A

Time—55 Minutes

Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. If $g(x) = \frac{1}{32}x^4 - 5x^2$, find $g'(4)$.

(A) –72
(B) –32
(C) –24
(D) 24
(E) 32

2. The domain of the function $f(x) = \sqrt{4 - x^2}$ is

(A) $x < -2$ or $x > 2$
(B) $x \leq -2$ or $x \geq 2$
(C) $-2 < x < 2$
(D) $-2 \leq x \leq 2$
(E) $x \leq 2$

3. $\lim_{x \to 5} \frac{x^2 - 25}{x - 5}$ is

(A) 0
(B) 10
(C) –10
(D) 5
(E) The limit does not exist.

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4. If $f(x) = \frac{x^5 - x + 2}{x^3 + 7}$, find $f'(x)$.

(A) $\frac{(5x^4 - 1)}{(3x^2)}$

(B) $\frac{(5x^4 - 1) - (3x^2)}{(x^3 + 7)}$

(C) $\frac{(x^3 + 7)(5x^4 - 1) - (x^5 - x + 2)(3x^2)}{(x^3 + 7)}$

(D) $\frac{(x^5 - x + 2)(3x^2) - (x^3 + 7)(5x^4 - 1)}{(x^3 + 7)^2}$

(E) $\frac{(x^3 + 7)(5x^4 - 1) - (x^5 - x + 2)(3x^2)}{(x^3 + 7)^2}$

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5. Evaluate $\lim_{h \to 0} \dfrac{5\left(\frac{1}{2}+h\right)^4 - 5\left(\frac{1}{2}\right)^4}{h}$.

(A) $\frac{5}{2}$

(B) $\frac{5}{16}$

(C) 40

(D) 160

(E) The limit does not exist.

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6. $\int x\sqrt{3x}\, dx =$

(A) $\frac{2\sqrt{3}}{5}x^{\frac{5}{2}} + C$

(B) $\frac{5\sqrt{3}}{2}x^{\frac{5}{2}} + C$

(C) $\frac{\sqrt{3}}{2}x^{\frac{1}{2}} + C$

(D) $2\sqrt{3x} + C$

(E) $\frac{5\sqrt{3}}{2}x^{\frac{3}{2}} + C$

---

7. Find k so that $f(x) = \begin{cases} \dfrac{x^2 - 16}{x - 4}; & x \neq 4 \\ k & ; x = 4 \end{cases}$ is continuous for all x.

(A) All real values of k make $f(x)$ continuous for all x.
(B) 0
(C) 16
(D) 8
(E) There is no real value of k that makes $f(x)$ continuous for all x.

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8. Which of the following integrals correctly gives the area of the region consisting of all points above the x-axis and below the curve $y = 8 + 2x - x^2$?

(A) $\int_{-2}^{4} (x^2 - 2x - 8)\, dx$

(B) $\int_{-4}^{2} (8 + 2x - x^2)\, dx$

(C) $\int_{-2}^{4} (8 + 2x - x^2)\, dx$

(D) $\int_{-4}^{2} (x^2 - 2x - 8)\, dx$

(E) $\int_{2}^{4} (8 + 2x - x^2)\, dx$

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9. If $f(x) = x^2 \cos 2x$, find $f'(x)$.

(A) $2x \sin 2x$
(B) $-2x \cos 2x + 2x^2 \sin 2x$
(C) $-4x \sin 2x$
(D) $2x \cos 2x - 2x^2 \sin 2x$
(E) $2x - 2 \sin 2x$

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10. An equation of the line tangent to $y = 4x^3 - 7x^2$ at $x = 3$ is

(A) $y + 45 = 66(x + 3)$

(B) $y - 45 = 66(x - 3)$

(C) $y = 66x$

(D) $y = 66(x - 3)$

(E) $y + 45 = \frac{-1}{66}(x - 3)$

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11. $\int_0^{\frac{1}{2}} \frac{2}{\sqrt{1-x^2}}\,dx =$

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{3}$

(C) $-\frac{\pi}{3}$

(D) $\frac{2\pi}{3}$

(E) $-\frac{2\pi}{3}$

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12. Find a positive value c, for x, that satisfies the conclusion of the Mean Value Theorem for Derivatives for $f(x) = 3x^2 - 5x + 1$ on the interval [2, 5].

(A) 1

(B) $\frac{13}{6}$

(C) $\frac{11}{6}$

(D) $\frac{23}{6}$

(E) $\frac{7}{2}$

---

13. Given $f(x) = 2x^2 - 7x - 10$, find the absolute maximum of $f(x)$ on $[-1, 3]$.

(A) -1

(B) $\frac{7}{4}$

(C) -13

(D) $-\frac{129}{8}$

(E) 0

---

14. Find $\frac{dy}{dx}$ if $x^3y + xy^3 = -10$.

(A) $(3x^2 + 3xy^2)$

(B) $-(3x^2 + 3xy^2)$

(C) $\frac{(3x^2y + y^3)}{(3xy^2 + x^3)}$

(D) $-\frac{(3x^2y + y^3)}{(3xy^2 + x^3)}$

(E) $-\frac{(x^2y + y^3)}{(xy^2 + x^3)}$

---

15. If $f(x) = \sqrt{1+\sqrt{x}}$, find $f'(x)$.

(A) $\dfrac{-1}{4\sqrt{x}\sqrt{1+\sqrt{x}}}$

(B) $\dfrac{1}{2\sqrt{x}\sqrt{1+\sqrt{x}}}$

(C) $\dfrac{1}{4\sqrt{1+\sqrt{x}}}$

(D) $\dfrac{1}{4\sqrt{x}\sqrt{1+\sqrt{x}}}$

(E) $\dfrac{-1}{2\sqrt{x}\sqrt{1+\sqrt{x}}}$

---

16. $\int 7xe^{3x^2}\,dx =$

(A) $\dfrac{1}{42}e^{3x^2} + C$

(B) $\dfrac{6}{7}e^{3x^2} + C$

(C) $\dfrac{7}{6}e^{3x^2} + C$

(D) $7e^{3x^2} + C$

(E) $42e^{3x^2} + C$

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17. Find the equation of the tangent line to $9x^2 + 16y^2 = 52$ through $(2, -1)$.

(A) $-9x + 8y - 26 = 0$
(B) $9x - 8y - 26 = 0$
(C) $9x - 8y - 106 = 0$
(D) $8x + 9y - 17 = 0$
(E) $9x + 16y - 2 = 0$

---

18. A particle's position is given by $s = t^3 - 6t^2 + 9t$. What is its acceleration at time $t = 4$?

(A) 0
(B) 9
(C) –9
(D) –12
(E) 12

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19. If $f(x) = 3^{\pi x}$, then $f'(x) =$

(A) $\dfrac{3^{\pi x}}{\pi \ln 3}$

(B) $\dfrac{3^{\pi x}}{\ln 3}$

(C) $\dfrac{3^{\pi x}}{\pi}$

(D) $\pi(3^{\pi x - 1})$

(E) $\pi \ln 3(3^{\pi x})$

---

20. The average value of $f(x) = \dfrac{1}{x}$ from $x = 1$ to $x = e$ is

(A) $\dfrac{1}{e+1}$

(B) $\dfrac{1}{1-e}$

(C) $e - 1$

(D) $1 - \dfrac{1}{e^2}$

(E) $\dfrac{1}{e-1}$

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21. If $f(x) = \sin^2 x$, find $f'''(x)$.

(A) $-\sin^2 x$
(B) $2 \cos 2x$
(C) $\cos 2x$
(D) $-4 \sin 2x$
(E) $-\sin 2x$

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22. Find the slope of the normal line to $y = x + \cos xy$ at (0, 1).

(A) 1
(B) –1
(C) 0
(D) 2
(E) Undefined

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23. $\int e^{x}(e^{3x})\,dx =$

(A) $\frac{1}{3}e^{3x} + C$

(B) $\frac{1}{4}e^{4x} + C$

(C) $\frac{1}{4}e^{5x} + C$

(D) $4e^{4x} + C$

(E) $4e^{5x} + C$

---

24. $\lim_{x \to 0} \frac{\tan^{3}(2x)}{x^{3}} =$

(A) –8
(B) –2
(C) 2
(D) 8
(E) The limit does not exist.

---

25. A solid is generated when the region in the first quadrant bounded by the graph of $y = 1 + \sin^2 x$, the line $x = \frac{\pi}{2}$, the x-axis, and the y-axis is revolved about the x-axis. Its volume is found by evaluating which of the following integrals?

(A) $\pi \int_0^1 (1 + \sin^4 x)\, dx$

(B) $\pi \int_0^1 (1 + \sin^2 x)^2\, dx$

(C) $\pi \int_0^{\frac{\pi}{2}} (1 + \sin^4 x)\, dx$

(D) $\pi \int_0^{\frac{\pi}{2}} (1 + \sin^2 x)^2\, dx$

(E) $\pi \int_0^{\frac{\pi}{2}} (1 + \sin^2 x)\, dx$

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26. If $y = \left(\frac{x^3 - 2}{2x^5 - 1}\right)^4$, find $\frac{dy}{dx}$ at $x = 1$.

(A) –52
(B) –28
(C) –13
(D) 13
(E) 52

---

27. $\int x\sqrt{5-x}\,dx =$

(A) $-\frac{10}{3}(5-x)^{\frac{3}{2}}$

(B) $\sqrt{\frac{5x^2}{2}-\frac{x^3}{3}}+C$

(C) $\frac{10}{3}\sqrt{\frac{5x^2}{2}-\frac{x^3}{3}}+C$

(D) $10(5-x)^{\frac{1}{2}}+\frac{2}{3}(5-x)^{\frac{3}{2}}+C$

(E) $-\frac{10}{3}(5-x)^{\frac{3}{2}}+\frac{2}{5}(5-x)^{\frac{5}{2}}+C$

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28. If $\frac{dy}{dx}=\frac{x^3+1}{y}$ and $y = 2$ when $x = 1$, then, when $x = 2$, $y =$

(A) $\sqrt{\frac{27}{2}}$

(B) $\sqrt{\frac{27}{8}}$

(C) $\pm\sqrt{\frac{27}{8}}$

(D) $\pm\frac{3}{2}$

(E) $\pm\sqrt{\frac{27}{2}}$

END OF PART A, SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS AB

SECTION I, Part B

Time—50 Minutes

Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

1. The **exact** numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

2. Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

29. The graph of $y = 5x^4 - x^5$ has an inflection point (or points) at

(A) $x = 0$ only
(B) $x = 3$ only
(C) $x = 0, 3$
(D) $x = -3$ only
(E) $x = 0, -3$

GO ON TO THE NEXT PAGE.

30. The average value of $f(x) = e^{4x^2}$ on the interval $\left[-\frac{1}{4}, \frac{1}{4}\right]$ is

(A) 0.272
(B) 0.545
(C) 1.090
(D) 2.180
(E) 4.360

---

31. $\int_0^1 \tan x \, dx =$

(A) 0

(B) $\frac{\tan^2 1}{2}$

(C) ln(cos(1))

(D) ln(sec(1))

(E) ln(sec(1)) – 1

---

32. $\frac{d}{dx}\int_0^{x^2} \sin^2 t \, dt =$

(A) $x^2 \sin^2(x^2)$
(B) $2x \sin^2(x^2)$
(C) $\sin^2(x^2)$
(D) $x^2 \cos^2(x^2)$
(E) $2x \cos^2(x^2)$

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33. Find the value(s) of $\frac{dy}{dx}$ of $x^2y + y^2 = 5$ at $y = 1$.

(A) $-\frac{3}{2}$ only
(B) $-\frac{2}{3}$ only
(C) $\frac{2}{3}$ only
(D) $\pm\frac{2}{3}$
(E) $\pm\frac{3}{2}$

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34. The graph of $y = x^3 - 2x^2 - 5x + 2$ has a local maximum at

(A) (2.120, 0)
(B) (2.120, –8.061)
(C) (–0.786, 0)
(D) (–0.786, 4.209)
(E) (0.666, –1.926)

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35. Approximate $\int_0^1 \sin^2 x \, dx$ using the Trapezoid Rule with $n = 4$, to three decimal places.

(A) 0.277
(B) 0.273
(C) 0.555
(D) 1.109
(E) 2.219

---

36. The volume generated by revolving about the x-axis the region above the curve $y = x^3$, below the line $y = 1$, and between $x = 0$ and $x = 1$ is

(A) $\frac{\pi}{42}$

(B) 0.143π

(C) $\frac{\pi}{7}$

(D) 0.643π

(E) $\frac{6\pi}{7}$

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37. A 20-foot ladder slides down a wall at 5 ft/sec. At what speed is the bottom sliding out when the top is 10 feet from the floor (in ft/sec)?

(A) 0.346
(B) 2.887
(C) 0.224
(D) 5.774
(E) 4.472

---

38. $\int \frac{\ln x}{3x}\, dx =$

(A) $6 \ln^2|x| + C$

(B) $\frac{1}{6} \ln(\ln|x|) + C$

(C) $\frac{1}{3} \ln^2|x| + C$

(D) $\frac{1}{6} \ln^2|x| + C$

(E) $\frac{1}{3} \ln|x| + C$

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39. Find two non-negative numbers x and y whose sum is 100 and for which x^2y is a maximum.

(A) $x = 33.333$ and $y = 33.333$
(B) $x = 50$ and $y = 50$
(C) $x = 33.333$ and $y = 66.667$
(D) $x = 100$ and $y = 0$
(E) $x = 66.667$ and $y = 33.333$

GO ON TO THE NEXT PAGE.

40. Find the distance traveled (to three decimal places) from $t = 1$ to $t = 5$ seconds, for a particle whose velocity is given by $v(t) = t + \ln t$.

(A) 6.000
(B) 1.609
(C) 16.047
(D) 0.800
(E) 148.413

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41. $\int \sin^5(2x)\cos(2x)\,dx =$

(A) $\dfrac{\sin^6 2x}{12} + C$

(B) $\dfrac{\sin^6 2x}{6} + C$

(C) $\dfrac{\sin^6 2x}{3} + C$

(D) $\dfrac{\cos^5 2x}{3} + C$

(E) $\dfrac{\cos^5 2x}{6} + C$

---

42. The volume of a cube is increasing at a rate proportional to its volume at any time t. If the volume is 8 ft^3 originally, and 12 ft^3 after 5 seconds, what is its volume at $t = 12$ seconds?

(A) 21.169
(B) 22.941
(C) 16.000
(D) 28.800
(E) 17.600

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43. If $f(x) = \left(1 + \frac{x}{20}\right)^5$, find $f''(40)$.

(A) 0.068
(B) 1.350
(C) 5.400
(D) 6.750
(E) 540.000

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44. A particle's height at a time $t \geq 0$ is given by $h(t) = 100t - 16t^2$. What is its maximum height?

(A) 312.500
(B) 156.250
(C) 78.125
(D) 6.250
(E) 3.125

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45. If $f(x)$ is continuous and differentiable and $f(x) = \begin{cases} ax^4 + 5x; & x \leq 2 \\ bx^2 - 3x; & x > 2 \end{cases}$, then $b =$

(A) 0.5
(B) 0
(C) 2
(D) 6
(E) There is no value of b.

STOP

END OF PART B, SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART B ONLY.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

SECTION II
GENERAL INSTRUCTIONS

You may wish to look over the problems before starting to work on them, since it is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

- You should write all work for each part of each problem in the space provided for that part in the booklet. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. You will be graded on the correctness and completeness of your methods as well as your answers. Correct answers without supporting work may not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons and that you clearly identify functions, graphs, tables, or other objects you use.
- You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt (X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

SECTION II, PART A
Time—30 minutes
Number of problems—2

A graphing calculator is required for some problems or parts of problems.

During the timed portion for Part A, you may work only on the problems in Part A.

On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

Section II

1. The temperature on New Year's Day in Hinterland was given by $T(H) = -A - B\cos\left(\frac{\pi H}{12}\right)$, where T is the temperature in degrees Fahrenheit and H is the number of hours from midnight $(0 \leq H < 24)$.

   (a) The initial temperature at midnight was $-15°$ F and at noon of New Year's Day was $5°$ F. Find A and B.

   (b) Find the average temperature for the first 10 hours.

   (c) Use the Trapezoid Rule with 4 equal subdivisions to estimate $\int_6^8 T(H)\,dH$.

   (d) Find an expression for the rate that the temperature is changing with respect to H.

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2. Sea grass grows on a lake. The rate of growth of the grass is $\frac{dG}{dt} = kG$, where k is a constant.

   (a) Find an expression for G, the amount of grass in the lake (in tons), in terms of t, the number of years, if the amount of grass is 100 tons initially and 120 tons after one year.

   (b) In how many years will the amount of grass available be 300 tons?

   (c) If fish are now introduced into the lake and consume a consistent 80 tons/year of sea grass, how long will it take for the lake to be completely free of sea grass?

GO ON TO THE NEXT PAGE.

SECTION II, PART B
Time—1 hour
Number of problems—4

No calculator is allowed for these problems.

During the timed portion for Part B, you may continue to work on the problems in Part A without the use of any calculator.

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3. Consider the curve defined by $y = x^4 + 4x^3$.
   (a) Find the equation of the tangent line to the curve at $x = -1$.
   (b) Find the coordinates of the absolute minimum.
   (c) Find the coordinates of the point(s) of inflection.

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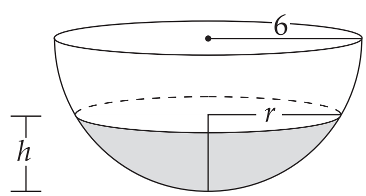
4. Water is being poured into a hemispherical bowl of radius 6 inches at the rate of 4 in^3/sec.



   (a) Given that the volume of the water in the spherical segment shown above is $V = \pi h^2 \left(R - \frac{h}{3}\right)$, where R is the radius of the *sphere,* find the rate that the water level is rising when the water is 2 inches deep.
   (b) Find an expression for r, the radius of the *surface of the spherical segment* of water, in terms of h.
   (c) How fast is the circular area of the surface of the spherical segment of water growing (in in^2/sec) when the water is 2 inches deep?

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5. Let R be the region in the first quadrant bounded by $y^2 = x$ and $x^2 = y$.

   (a) Find the area of region R.

   (b) Find the volume of the solid generated when R is revolved about the x-axis.

   (c) The section of a certain solid cut by any plane perpendicular to the x-axis is a circle with the endpoints of its diameter lying on the parabolas $y^2 = x$ and $x^2 = y$. Find the volume of the solid.

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6. An object moves with velocity $v(t) = t^2 - 8t + 7$.

   (a) Write a polynomial expression for the position of the particle at any time $t \geq 0$.

   (b) At what time(s) is the particle changing direction?

   (c) Find the total distance traveled by the particle from time $t = 0$ to $t = 4$.

STOP

END OF EXAM