CALCULUS AB

SECTION I, Part A

Time—55 Minutes

Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. If $f(x) = 5x^{\frac{4}{3}}$, then $f'(8) =$

(A) 10

(B) $\frac{40}{3}$

(C) 40

(D) 80

(E) $\frac{160}{3}$

2. $\lim_{x\to\infty} \dfrac{5x^2 - 3x + 1}{4x^2 + 2x + 5}$ is

(A) 0

(B) $\dfrac{4}{5}$

(C) $\dfrac{3}{11}$

(D) $\dfrac{5}{4}$

(E) ∞

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3. If $f(x) = \dfrac{3x^2 + x}{3x^2 - x}$, then $f'(x)$ is

(A) 1

(B) $\dfrac{6x^2 + 1}{6x^2 - 1}$

(C) $\dfrac{-6}{(3x - 1)^2}$

(D) $\dfrac{-2x^2}{(x^2 - x)^2}$

(E) $\dfrac{36x^3 - 2x}{(x^2 - x)^2}$

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4. If the function f is continuous for all real numbers and if $f(x) = \dfrac{x^2 - 7x + 12}{x - 4}$ when $x \neq 4$, then $f(4) =$

(A) 1

(B) $\dfrac{8}{7}$

(C) -1

(D) 0

(E) undefined

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5. If $x^2 - 2xy + 3y^2 = 8$, then $\dfrac{dy}{dx} =$

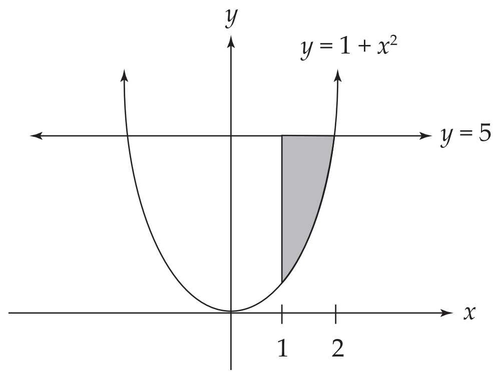
(A) $\dfrac{8 + 2y - 2x}{6y - 2x}$

(B) $\dfrac{3y - x}{y - x}$

(C) $\dfrac{2x - 2y}{6y - 2x}$

(D) $\dfrac{1}{3}$

(E) $\dfrac{y - x}{3y - x}$



6. Which of the following integrals correctly corresponds to the area of the shaded region in the figure above ?

(A) $\int_1^2 (x^2 - 4)\, dx$

(B) $\int_1^2 (4 - x^2)\, dx$

(C) $\int_1^5 (x^2 - 4)\, dx$

(D) $\int_1^2 (x^2 + 4)\, dx$

(E) $\int_1^5 (4 - x^2)\, dx$

7. If $f(x) = \sec x + \csc x$, then $f'(x) =$

(A) 0
(B) $\sec^2 x + \csc^2 x$
(C) $\csc x - \sec x$
(D) $\sec x \tan x + \csc x \cot x$
(E) $\sec x \tan x - \csc x \cot x$

8. An equation of the line normal to the graph of $y = \sqrt{(3x^2 + 2x)}$ at (2, 4) is

(A) $-4x + y = 20$
(B) $4x + 7y = 20$
(C) $-7x + 4y = 2$
(D) $7x + 4y = 30$
(E) $4x + 7y = 36$

9. $\int_{-1}^{1} \frac{4}{1 + x^2}\, dx =$

(A) 0
(B) π
(C) 1
(D) 2π
(E) 2

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10. If $f(x) = \cos^2 x$, then $f''(\pi) =$

(A) -2
(B) 0
(C) 1
(D) 2
(E) 2π

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11. If $f(x) = \dfrac{5}{x^2 + 1}$ and $g(x) = 3x$, then $g(f(2)) =$

(A) -3

(B) $\dfrac{5}{37}$

(C) 3

(D) 5

(E) $\dfrac{37}{5}$

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12. $\int x\sqrt{5x^2 - 4}\; dx =$

(A) $\dfrac{1}{10}(5x^2 - 4)^{\frac{3}{2}} + C$

(B) $\dfrac{1}{15}(5x^2 - 4)^{\frac{3}{2}} + C$

(C) $-\dfrac{1}{5}(5x^2 - 4)^{-\frac{1}{2}} + C$

(D) $\dfrac{20}{3}(5x^2 - 4)^{\frac{3}{2}} + C$

(E) $\dfrac{3}{20}(5x^2 - 4)^{\frac{3}{2}} + C$

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13. The slope of the line tangent to the graph of $3x^2 + 5 \ln y = 12$ at (2, 1) is

(A) $-\frac{12}{5}$

(B) $\frac{12}{5}$

(C) $\frac{5}{12}$

(D) 12

(E) –7

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14. The equation $y = 2 - 3 \sin \frac{\pi}{4}(x - 1)$ has a fundamental period of

(A) $\frac{1}{8}$

(B) $\frac{\pi}{4}$

(C) $\frac{4}{\pi}$

(D) 8

(E) 2π

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15. If $f(x)=\begin{cases} x^2+5 \text{ if } x<2 \\ 7x-5 \text{ if } x\geq 2 \end{cases}$, for all real numbers x, which of the following must be true?

I. $f(x)$ is continuous everywhere.
II. $f(x)$ is differentiable everywhere.
III. $f(x)$ has a local minimum at $x = 2$.

(A) I only
(B) I and II only
(C) II and III only
(D) I and III only
(E) I, II, and III

16. For what value of x does the function $f(x) = x^3 - 9x^2 - 120x + 6$ have a local minimum?

(A) 10
(B) 4
(C) 3
(D) –4
(E) –10

17. The acceleration of a particle moving along the x-axis at time t is given by $a(t) = 4t - 12$. If the velocity is 10 when $t = 0$ and the position is 4 when $t = 0$, then the particle is changing direction at

(A) $t = 1$

(B) $t = 3$

(C) $t = 5$

(D) $t = 1$ and $t = 5$

(E) $t = 1$, $t = 3$, and $t = 5$

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18. The average value of the function $f(x) = (x - 1)^2$ on the interval from $x = 1$ to $x = 5$ is

(A) $-\dfrac{16}{3}$

(B) $\dfrac{16}{3}$

(C) $\dfrac{64}{3}$

(D) $\dfrac{66}{3}$

(E) $\dfrac{256}{3}$

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19. $\int (e^{3\ln x} + e^{3x})\,dx =$

(A) $3 + \dfrac{e^{3x}}{3} + C$

(B) $\dfrac{x^4}{4} + 3e^{3x} + C$

(C) $\dfrac{e^{x^4}}{4} + 3e^{3x} + C$

(D) $\dfrac{e^{x^4}}{4} + \dfrac{e^{3x}}{3} + C$

(E) $\dfrac{x^4}{4} + \dfrac{e^{3x}}{3} + C$

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20. If $f(x) = (x^2 + x + 11)\sqrt{(x^3 + 5x + 121)}$, then $f'(0) =$

(A) $\dfrac{5}{2}$

(B) $\dfrac{27}{2}$

(C) 22

(D) $22 + \dfrac{2}{\sqrt{5}}$

(E) $\dfrac{247}{2}$

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21. If $f(x) = 5^{3x}$, then $f'(x) =$

(A) $5^{3x}(\ln 125)$

(B) $\dfrac{5^{3x}}{3 \ln 5}$

(C) $3(5^{2x})$

(D) $3(5^{3x})$

(E) $3x(5^{3x-1})$

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22. A solid is generated when the region in the first quadrant enclosed by the graph of $y = (x^2 + 1)^3$, the line $x = 1$, the x-axis, and the y-axis is revolved about the x-axis. Its volume is found by evaluating which of the following integrals?

(A) $\pi\int_1^8 (x^2+1)^3\,dx$

(B) $\pi\int_1^8 (x^2+1)^6\,dx$

(C) $\pi\int_0^1 (x^2+1)^3\,dx$

(D) $\pi\int_0^1 (x^2+1)^6\,dx$

(E) $2\pi\int_0^1 (x^2+1)^6\,dx$

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23. $\lim_{x \to 0} 4 \dfrac{\sin x \cos x - \sin x}{x^2} =$

(A) 2

(B) $\dfrac{40}{3}$

(C) ∞

(D) 0

(E) undefined

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24. If $\dfrac{dy}{dx} = \dfrac{(3x^2 + 2)}{y}$ and $y = 4$ when $x = 2$, then when $x = 3$, $y =$

(A) 18

(B) $\pm\sqrt{66}$

(C) 58

(D) $\pm\sqrt{74}$

(E) $\pm\sqrt{58}$

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25. $\int \frac{dx}{9+x^2} =$

(A) $3\tan^{-1}\left(\frac{x}{3}\right)+C$

(B) $\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)+C$

(C) $\frac{1}{9}\tan^{-1}\left(\frac{x}{3}\right)+C$

(D) $\frac{1}{3}\tan^{-1}(x)+C$

(E) $\frac{1}{9}\tan^{-1}(x)+C$

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26. If $f(x) = \cos^3(x + 1)$, then $f'(\pi) =$

(A) $-3\cos^2(\pi + 1)\sin(\pi + 1)$
(B) $3\cos^2(\pi + 1)$
(C) $3\cos^2(\pi + 1)\sin(\pi + 1)$
(D) $3\pi\cos^2(\pi + 1)$
(E) 0

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27. $\int x\sqrt{x+3}\,dx =$

(A) $\frac{2}{3}(x)^{\frac{3}{2}}+6(x)^{\frac{1}{2}}+C$

(B) $\frac{2(x+3)^{\frac{3}{2}}}{3}+C$

(C) $\frac{2}{5}(x+3)^{\frac{5}{2}}-2(x+3)^{\frac{3}{2}}+C$

(D) $\frac{3(x+3)^{\frac{3}{2}}}{2}+C$

(E) $\frac{4x^2(x+3)^{\frac{3}{2}}}{3}+C$

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28. If $f(x) = \ln(\ln(1-x))$, then $f'(x) =$

(A) $-\frac{1}{\ln(1-x)}$

(B) $\frac{1}{(1-x)\ln(1-x)}$

(C) $\frac{1}{(1-x)^2}$

(D) $-\frac{1}{(1-x)\ln(1-x)}$

(E) $-\frac{1}{\ln(1-x)^2}$

END OF PART A, SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS AB

SECTION I, Part B

Time—50 Minutes

Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

1. The **exact** numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

2. Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

29. $\int_0^{\frac{\pi}{4}} \sin x \, dx + \int_{-\frac{\pi}{4}}^0 \cos x \, dx =$

(A) $-\sqrt{2}$

(B) -1

(C) 0

(D) 1

(E) $\sqrt{2}$

30. Boats A and B leave the same place at the same time. Boat A heads due north at 12 km/hr. Boat B heads due east at 18 km/hr. After 2.5 hours, how fast is the distance between the boats increasing (in km/hr)?

(A) 21.63
(B) 31.20
(C) 75.00
(D) 9.84
(E) 54.08

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31. $\lim_{h \to 0} \dfrac{\tan\left(\dfrac{\pi}{6} + h\right) - \tan\left(\dfrac{\pi}{6}\right)}{h} =$

(A) $\dfrac{\sqrt{3}}{3}$

(B) $\dfrac{4}{3}$

(C) $\sqrt{3}$

(D) 0

(E) $\dfrac{3}{4}$

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32. If $\int_{30}^{100} f(x)\,dx = A$ and $\int_{50}^{100} f(x)\,dx = B$, then $\int_{30}^{50} f(x)\,dx =$

(A) $A + B$
(B) $A - B$
(C) 0
(D) $B - A$
(E) 20

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33. If $f(x) = 3x^2 - x$, and $g(x) = f^{-1}(x)$, then $g'(10)$ could be

(A) 59

(B) $\dfrac{1}{59}$

(C) $\dfrac{1}{10}$

(D) 11

(E) $\dfrac{1}{11}$

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34. The graph of $y = x^3 - 5x^2 + 4x + 2$ has a local minimum at

(A) (0.46, 2.87)
(B) (0.46, 0)
(C) (2.87, –4.06)
(D) (4.06, 2.87)
(E) (1.66, –0.59)

35. The volume generated by revolving about the y-axis the region enclosed by the graphs $y = 9 - x^2$ and $y = 9 - 3x$, for $0 \leq x \leq 2$, is

(A) -8π
(B) 4π
(C) 8π
(D) 24π
(E) 48π

36. The average value of the function $f(x) = \ln^2 x$ on the interval [2, 4] is

(A) –1.204
(B) 1.204
(C) 2.159
(D) 2.408
(E) 8.636

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37. $\dfrac{d}{dx}\displaystyle\int_0^{3x} \cos(t)\, dt =$

(A) $\sin 3x$
(B) $-3 \sin 3x$
(C) $\cos 3x$
(D) $3 \sin 3x$
(E) $3 \cos 3x$

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38. If the definite integral $\int_1^3 (x^2+1)\,dx$ is approximated by using the Trapezoid Rule with $n = 4$, the error is

(A) 0

(B) $\frac{7}{3}$

(C) $\frac{1}{12}$

(D) $\frac{65}{6}$

(E) $\frac{97}{3}$

39. The radius of a sphere is increasing at a rate proportional to itself. If the radius is 4 initially, and the radius is 10 after two seconds, what will the radius be after three seconds?

(A) 62.50
(B) 13.00
(C) 15.81
(D) 16.00
(E) 25.00

40. Use differentials to approximate the change in the volume of a sphere when the radius is increased from 10 to 10.02 cm.

(A) 4,213.973
(B) 1,261.669
(C) 1,256.637
(D) 25.233
(E) 25.133

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41. $\int \ln 2x \, dx =$

(A) $\frac{\ln 2x}{x} + C$

(B) $\frac{\ln 2x}{2x} + C$

(C) $x \ln x - x + C$

(D) $x \ln 2x - x + C$

(E) $2x \ln 2x - 2x + C$

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42. If the function $f(x)$ is differentiable and $f(x) = \begin{cases} ax^3 - 6x; \text{ if } x \leq 1 \\ bx^2 + 4; \ x > 1 \end{cases}$, then $a =$

(A) 0
(B) 1
(C) –14
(D) –24
(E) 26

43. Two particles leave the origin at the same time and move along the y-axis with their respective positions determined by the functions $y_1 = \cos 2t$ and $y_2 = 4\sin t$ for $0 < t < 6$. For how many values of t do the particles have the same acceleration?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

44. Find the distance traveled (to three decimal places) in the first four seconds, for a particle whose velocity is given by $v(t) = 7e^{-t^2}$, where t stands for time.

(A) 0.976
(B) 6.204
(C) 6.359
(D) 12.720
(E) 7.000

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45. $\int \tan^6 x \sec^2 x\, dx =$

(A) $\dfrac{\tan^7 x}{7} + C$

(B) $\dfrac{\tan^7 x}{7} + \dfrac{\sec^3 x}{3} + C$

(C) $\dfrac{\tan^7 x \sec^3 x}{21} + C$

(D) $7\tan^7 x + C$

(E) $\dfrac{2}{7}\tan^7 x \sec x + C$

STOP

END OF PART B, SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART B ONLY.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

SECTION II
GENERAL INSTRUCTIONS

You may wish to look over the problems before starting to work on them, since it is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

- You should write all work for each part of each problem in the space provided for that part in the booklet. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. You will be graded on the correctness and completeness of your methods as well as your answers. Correct answers without supporting work may not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons and that you clearly identify functions, graphs, tables, or other objects you use.
- You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt (X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

SECTION II, PART A
Time—30 minutes
Number of problems—2

A graphing calculator is required for some problems or parts of problems.

During the timed portion for Part A, you may work only on the problems in Part A.

On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

GO ON TO THE NEXT PAGE.

1. A particle moves along the x-axis so that its acceleration at any time $t > 0$ is given by $a(t) = 12t - 18$. At time $t = 1$, the velocity of the particle is $v(1) = 0$ and the position is $x(1) = 9$.

 (a) Write an expression for the velocity of the particle $v(t)$.

 (b) At what values of t does the particle change direction?

 (c) Write an expression for the position $x(t)$ of the particle.

 (d) Find the total distance traveled by the particle from $t = \frac{3}{2}$ to $t = 6$.

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2. Let R be the region enclosed by the graphs of $y = 2 \ln x$ and $y = \frac{x}{2}$, and the lines $x = 2$ and $x = 8$.

 (a) Find the area of R.

 (b) Set up, <u>but do not integrate</u>, an integral expression, in terms of a single variable, for the volume of the solid generated when R is revolved about the x-axis.

 (c) Set up, <u>but do not integrate</u>, an integral expression, in terms of a single variable, for the volume of the solid generated when R is revolved about the line $x = -1$.

SECTION II, PART B
Time—1 hour
Number of problems—4

No calculator is allowed for these problems.

During the timed portion for Part B, you may continue to work on the problems in Part A without the use of any calculator.

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3. Consider the equation $x^2 - 2xy + 4y^2 = 64$.

 (a) Write an expression for the slope of the curve at any point (x, y).

 (b) Find the equation of the tangent lines to the curve at the point $x = 2$.

 (c) Find $\frac{d^2y}{dx^2}$ at $(0, 4)$.

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4. Water is draining at the rate of 48π ft^3/second from the vertex at the bottom of a conical tank whose diameter at its base is 40 feet and whose height is 60 feet.

 (a) Find an expression for the volume of water in the tank, in terms of its radius, at the surface of the water.

 (b) At what rate is the radius of the water in the tank shrinking when the radius is 16 feet?

 (c) How fast is the height of the water in the tank dropping at the instant that the radius is 16 feet?

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5. Let f be the function given by $f(x) = 2x^4 - 4x^2 + 1$.

   (a) Find an equation of the line tangent to the graph at $(-2, 17)$.

   (b) Find the x- and y-coordinates of the relative maxima and relative minima. Verify your answer.

   (c) Find the x- and y-coordinates of the points of inflection. Verify your answer.

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6. Let $F(x) = \int_0^x \left[\cos\left(\frac{t}{2}\right) + \left(\frac{3}{2}\right)\right] dt$ on the closed interval $[0, 4\pi]$.

   (a) Approximate $F(2\pi)$ using four inscribed rectangles.

   (b) Find $F'(2\pi)$.

   (c) Find the average value of $F'(x)$ on the interval $[0, 4\pi]$.

STOP

END OF EXAM