

Chapter 17

General and Partial Fraction Integration Drill

GENERAL AND PARTIAL FRACTION INTEGRATION DRILL

1. Find $f(x)$ if $f(0) = 8$ and $f'(x) = 1 - 6x$.
- (A) $8 + x + 3x^2$
(B) $8 + x - 3x^2$
(C) $6 - x + 3x^2$
(D) $-8 + x + 3x^2$
(E) $1 - x - 3x^2$
2. If $f'(x) = 8x^3 + 12x + 3$, find $f(x)$ if $f(1) = 6$.
- (A) $2x^4 + 6x^2 + 3x - 5$
(B) $5x^4 + 3x^2 + 2x + 5$
(C) $-4x^4 + 6x^2 + 3x - 5$
(D) $5x^4 + 6x^2 + 3x + 5$
(E) $2x^4 - 6x^2 - 3x - 5$
3. If $f\left(\frac{\pi}{3}\right) = 4$, and $f'(x) = 2\cos x + \sec^2 x$, then find $f(x)$.
- (A) $-2\cos x + \tan x + 4$
(B) $2\sin x - \tan x + 4$
(C) $2 \sin x + \tan x + 4 - 2\sqrt{3}$
(D) $-2 \sin x + 2 \sec x + 4 + 2\sqrt{3}$
(E) $2 \sin x + 2 \tan x + 4 + 2\sqrt{3}$
4. If $f''(x) = 24x^2 + 2x + 10$, $f(1) = 5$, and $f'(1) = -3$, then $f(x) =$
- (A) $2x^4 + x^3 + 5x^2 - 22x - \frac{59}{3}$
(B) $2x^4 - \frac{x^3}{3} - 5x^2 - 22x + \frac{59}{3}$
(C) $-2x^4 + \frac{x^3}{3} + 5x^2 - 22x + \frac{59}{3}$
(D) $2x^4 - 4x^3 + 5x^2 - 22x + \frac{59}{3}$
(E) $2x^4 + \frac{x^3}{3} + 5x^2 - 22x + \frac{59}{3}$
5. Given $f(0) = 2$, $f'(0) = 1$, $f''(x) = 4 - 6x - 40x^3$, what is $f(x)$?
- (A) $2 - x - 2x^2 - x^3 + 2x^5$
(B) $2 + 2x + 3x^2 - x^3 - 6x^5$
(C) $-2 + x + 2x^2 - x^3 - 2x^5$
(D) $2 + x + 2x^2 - x^3 - 2x^5$
(E) $2 + x + 2x^2 + x^3 + 6x^5$
6. Given $f''(x) = \sin x + \cos x$, $f(0) = 3$, and $f'(0) = 4$, then what is $f(x)$?
- (A) $\sin x + \cos x + 5x + 4$
(B) $\cos x - \sin x + 5x + 4$
(C) $\sin x - \cos x - 5x - 4$
(D) $\sin x - \cos x + 5x + 4$
(E) $-\sin x - \cos x + 5x + 4$
7. If $f''(x) = 2 - 12x$, $f(0) = 9$, and $f(2) = 15$, then find $f(x)$.
- (A) $2x^3 - x^2 + 9x + 9$
(B) $-2x^3 + x^2 - 9x + 9$
(C) $2x^3 + x^2 + 9x + 9$
(D) $-2x^3 - x^2 + 9x - 9$
(E) $-2x^3 + x^2 + 9x + 9$

8. Find $f(x)$ if $f''(x) = 20x^3 + 12x^2 + 4$, $f(0) = 8$, and $f(1) = 5$.

- (A) $-x^5 + x^4 + 2x^2 - 5x + 2$
- (B) $x^5 + x^4 + 2x^2 - 7x + 8$
- (C) $6x^5 + 5x^4 + 2x^2 - 7x + 8$
- (D) $-x^5 + x^4 + 2x^2 - 7x + 8$
- (E) $x^5 + x^4 + 2x^2 - 5x + 2$

9. Find $f(x)$ if $f'(0) = 2$, $f(1) = 1$, and $f''(x) = 20x^3 + 12x^2 + 4$.

- (A) $x^5 + x^4 + 2x^2 - 5x + 2$
- (B) $6x^5 + x^4 + 2x^2 - 5x + 2$
- (C) $x^5 - x^4 - 2x^2 - 5x - 2$
- (D) $-x^5 + x^4 + 2x^2 - 5x + 2$
- (E) $x^5 + 5x^4 + 3x^2 - 5x + 2$

10. Find $f(x)$ if $f''(x) = 48x^2 - 6x + 1$, $f(0) = 1$, and $f'(0) = 2$.

- (A) $x^4 - 4x^3 + \frac{1}{2}x^2 + 2x + 1$
- (B) $x^4 - \frac{1}{2}x^3 + x^2 + 2x + 1$
- (C) $4x^4 - x^3 + \frac{1}{2}x^2 + 2x + 1$
- (D) $-4x^4 + x^3 + \frac{1}{2}x^2 + 2x + 1$
- (E) $4x^4 + x^3 + 2x^2 + \frac{1}{2}x + 1$

11. $\int_1^4 (5 - 2x + 3x^2) dx =$

- (A) -63
- (B) -14
- (C) 2
- (D) 14
- (E) 63

12. $\int_0^2 x(2 + x^5) dx =$

- (A) -33
- (B) $-\frac{156}{7}$
- (C) 0
- (D) $\frac{156}{7}$
- (E) 33

13. $\int \frac{x^3 - 2\sqrt{x}}{x} dx =$

- (A) $x^4 - x^2 + C$
- (B) $x^{\frac{3}{2}} - x + C$
- (C) $\frac{2}{3}x^{\frac{1}{3}} - 4\sqrt{x} + C$
- (D) $\frac{1}{3}x^{\frac{1}{3}} - 4\sqrt{x} + C$
- (E) $x^{\frac{1}{3}} - 4\sqrt{x} + C$

14. $\int (3x^2 + 2x)(x^3 + x^2)^5 dx =$

- (A) $\frac{1}{6}(x^3 + x^2)^6 + C$
- (B) $\frac{1}{6}(3x^2 + 2x)^6 + C$
- (C) $(x^3 + x^2)^6 + C$
- (D) $(3x^2 + 2x)^6 + C$
- (E) $-\frac{1}{6}(x^3 + x^2)^6 + C$

$$15. \quad \frac{1}{2} \int \left(\frac{dx}{\sqrt{x}} \right) - 3 \int x^2 dx =$$

- (A) $x - x^3 + C$
(B) $x^2 - x^3 + C$
(C) $\sqrt{x} - x^3 + C$
(D) $2\sqrt{x} - x^3 + C$
(E) $\frac{1}{2}\sqrt{x} - x^3 + C$

$$16. \quad \int (x^2 + x - x^{-1} + 2x^{-2}) dx =$$

- (A) $2x + 1 + x^2 - 4x^3 + C$
(B) $-\frac{1}{3}x^3 - \frac{1}{2}x^2 + \ln x + 2x^1 + C$
(C) $\frac{1}{3}x^3 + \frac{1}{2}x^2 - \ln x - 2x^1 + C$
(D) $2x - 1 - x^2 + 4x^3 + C$
(E) $3x + C$

$$17. \quad \int (2x^2 + 3) dx =$$

- (A) $2x^3 + 3x + C$
(B) $x^3 + 3x + C$
(C) $\frac{2}{3}x^3 + 3x + C$
(D) $4x + C$
(E) $4x^3 + 3 + C$

$$18. \quad \int x(2x^2 + 7)^5 dx =$$

- (A) $\frac{(2x^2 + 7)^4}{4} + C$
(B) $\frac{(2x^2 + 7)^6}{24} + C$
(C) $(2x^2 + 7)^6 + C$
(D) $\frac{(2x^2 + 7)^6}{6} + C$
(E) $(2x^2 + 7)^4 + C$

$$19. \quad \int_0^3 (x^2 - x)(6x^3 - 9x^2) dx =$$

- (A) $\frac{725}{4}$
(B) $\frac{727}{4}$
(C) $\frac{729}{4}$
(D) $\frac{731}{4}$
(E) $\frac{733}{4}$

$$20. \quad \text{Find } \frac{d}{dx} \int_7^{x^3} (2t - 3t^3) dt =$$

- (A) $-x^3$
(B) $6x^5 - 9x^{11}$
(C) $2x^3 - 3x^6$
(D) $2x^3 - 3x^9$
(E) $6x^6 - 3x^{18}$

21. $\int_{-2}^2 \frac{1}{x^4} dx =$

- (A) 0
 (B) $\frac{1}{24}$
 (C) $\frac{1}{12}$
 (D) $\frac{1}{8}$
 (E) The integral diverges.

22. $\int \frac{x-9}{(x+5)(x-2)} dx =$

- (A) $-2 \int \frac{dx}{(x+5)} + \int \frac{dx}{(x-2)}$
 (B) $\int \frac{dx}{(x+5)} - 2 \int \frac{dx}{(x-2)}$
 (C) $2 \int \frac{dx}{(x+5)} + 2 \int \frac{dx}{(x-2)}$
 (D) $2 \int \frac{dx}{(x+5)} - \int \frac{dx}{(x-2)}$
 (E) $\int \frac{dx}{(x+5)} - \int \frac{dx}{(x-2)}$

23. $\int \frac{1}{(x+4)(x-1)} dx =$

- (A) $\frac{1}{5} \int \frac{dx}{(x+4)} - \frac{1}{5} \int \frac{dx}{(x-1)}$
 (B) $-\frac{1}{5} \int \frac{dx}{(x+4)} + \frac{1}{5} \int \frac{dx}{(x-1)}$
 (C) $-\frac{4}{5} \int \frac{dx}{(x+4)} + \frac{4}{5} \int \frac{dx}{(x-1)}$
 (D) $\frac{4}{5} \int \frac{dx}{(x+4)} - \frac{4}{5} \int \frac{dx}{(x-1)}$
 (E) $\frac{1}{5} \int \frac{dx}{(x+4)} + \frac{1}{5} \int \frac{dx}{(x-1)}$

24. $\int \frac{2}{x^2-1} dx =$

- (A) $-\ln|x-1| + \ln|x+1| + C$
 (B) $2|\ln x-1| - \ln|x+1| + C$
 (C) $\ln|x-1| - \ln|x+1| + C$
 (D) $\ln|x-1| - 2\ln|x+1| + C$
 (E) $2\ln|x-1| - 2\ln|x+1| + C$

25. $\int \frac{3}{x^2+x-2} dx =$

- (A) $-\int \frac{dx}{(x+2)} + \int \frac{dx}{(x-1)}$
 (B) $\int \frac{dx}{(x+2)} - \int \frac{dx}{(x-1)}$
 (C) $-3 \int \frac{dx}{(x+2)} + \int \frac{dx}{(x-1)}$
 (D) $3 \int \frac{dx}{(x+2)} - \int \frac{dx}{(x-1)}$
 (E) $\int \frac{dx}{(x+2)} + \int \frac{dx}{(x-1)}$

26. $\int \frac{5-x}{2x^2+x-1} dx =$

- (A) $-3 \int \frac{dx}{(2x-1)} + 2 \int \frac{dx}{(x+1)}$
 (B) $5 \int \frac{dx}{(2x-1)} - \int \frac{dx}{(x+1)}$
 (C) $2 \int \frac{dx}{(2x-1)} - 3 \int \frac{dx}{(x+1)}$
 (D) $-2 \int \frac{dx}{(2x-1)} - 3 \int \frac{dx}{(x+1)}$
 (E) $3 \int \frac{dx}{(2x-1)} - 2 \int \frac{dx}{(x+1)}$

27. $\int \frac{x^2 + 12x + 12}{x^3 - 4x} dx =$

- (A) $-3 \int \frac{dx}{x} - \int \frac{dx}{(x+2)} + 5 \int \frac{dx}{(x-2)}$
- (B) $-5 \int \frac{dx}{x} - 3 \int \frac{dx}{(x+2)} + \int \frac{dx}{(x-2)}$
- (C) $3 \int \frac{dx}{x} - \int \frac{dx}{(x+2)} - 5 \int \frac{dx}{(x-2)}$
- (D) $\int \frac{dx}{x} - 3 \int \frac{dx}{(x+2)} + 5 \int \frac{dx}{(x-2)}$
- (E) $-3 \int \frac{dx}{x} + \int \frac{dx}{(x+2)} - 5 \int \frac{dx}{(x-2)}$

28. $\int \frac{4x^2 + 2x - 1}{x^3 + x^2} dx =$

- (A) $-3 \int \frac{dx}{x} + \int \frac{dx}{x^2} - \int \frac{dx}{(x+1)}$
- (B) $\int \frac{dx}{x} - 3 \int \frac{dx}{x^2} + \int \frac{dx}{(x+1)}$
- (C) $-\int \frac{dx}{x} - \int \frac{dx}{x^2} + 3 \int \frac{dx}{(x+1)}$
- (D) $\int \frac{dx}{x} + 3 \int \frac{dx}{x^2} - \int \frac{dx}{(x+1)}$
- (E) $3 \int \frac{dx}{x} - \int \frac{dx}{x^2} + \int \frac{dx}{(x+1)}$

29. $\int \frac{x^2 - 1}{x^3 + x} dx =$

- (A) $\int \frac{dx}{x} - \int \frac{2x}{(x^2 + 1)}$
- (B) $-\int \frac{dx}{x} + \int \frac{2x}{(x^2 + 1)}$
- (C) $2 \int \frac{dx}{x} + \int \frac{2x}{(x^2 + 1)}$
- (D) $-2 \int \frac{dx}{x} - \int \frac{2x}{(x^2 + 1)}$
- (E) $-\int \frac{dx}{x} - \int \frac{2x}{(x^2 + 1)}$

30. $\int \frac{1}{x^2 + x} dx =$

- (A) $\ln x + \ln |x+1| + C$
- (B) $-\ln x + \ln |x+1| + C$
- (C) $\ln x - \ln |x+1| + C$
- (D) $-\ln x - \ln |x+1| + C$
- (E) $\ln x - \ln |x-1| + C$

31. $\int \frac{1}{2x^2 + x} dx =$

- (A) $-\ln x - \ln |2x+1| + C$
- (B) $\ln x + \ln |2x+1| + C$
- (C) $-\ln x + \ln |2x+1| + C$
- (D) $\ln x - \ln |2x+1| + C$
- (E) $\ln x - \ln |2x-1| + C$

32. $\int y(y^2 + 1)^5 dy =$

- (A) $(y^2 + 1)^6 + C$
- (B) $\frac{1}{4}(y^2 + 1)^4 + C$
- (C) $\frac{1}{5}(y^2 + 1)^5 + C$
- (D) $\frac{1}{6}(y^2 + 1)^6 + C$
- (E) $\frac{1}{12}(y^2 + 1)^6 + C$

33. $\int \frac{\sqrt{x} - 2x^2}{x} dx =$

- (A) $\sqrt{x} - x^2 + C$
- (B) $\frac{1}{2}\sqrt{x} - x^2 + C$
- (C) $\frac{1}{3}\sqrt{x} - x^3 + C$
- (D) $2\sqrt{x} - x^2 + C$
- (E) $3\sqrt{x} - x^3 + C$

34. $\int_{-4}^{-2} (x^3 + 5x^2 - 7) dx =$

- (A) 19.333
- (B) 24.667
- (C) 33
- (D) 38.333
- (E) 46

35. If the series $\sum_{n=1}^{\infty} \frac{6}{4^n}$ converges, then which of the following series will not converge?

- (A) $\sum_{n=1}^{\infty} \frac{1}{2+4^n}$
- (B) $\sum_{n=1}^{\infty} \frac{6}{4n}$
- (C) $\sum_{n=1}^{\infty} \frac{1}{4^n}$
- (D) $\sum_{n=1}^{\infty} \frac{6n}{4^n}$
- (E) $\sum_{n=1}^{\infty} \frac{6}{2+4^n}$

36. To what value does the series $\sum_{n=1}^{\infty} (n+1)2^n$ converge?

- (A) 0
- (B) $\frac{1}{2}$
- (C) 1
- (D) 2
- (E) The series diverges.

37. A particle's acceleration is given by $a(t) = 6t^2 - 4$. Its initial position is 1 and its velocity at $t = 2$ is 9. What is the position of the particle at $t = 4$?

- (A) 113
- (B) 110
- (C) 101
- (D) 100
- (E) 92

38. Find the average value of $f(x) = x^3 - 2$ from $x = 1$ to $x = 3$.

- (A) 2
- (B) 4
- (C) 8
- (D) 16
- (E) 32