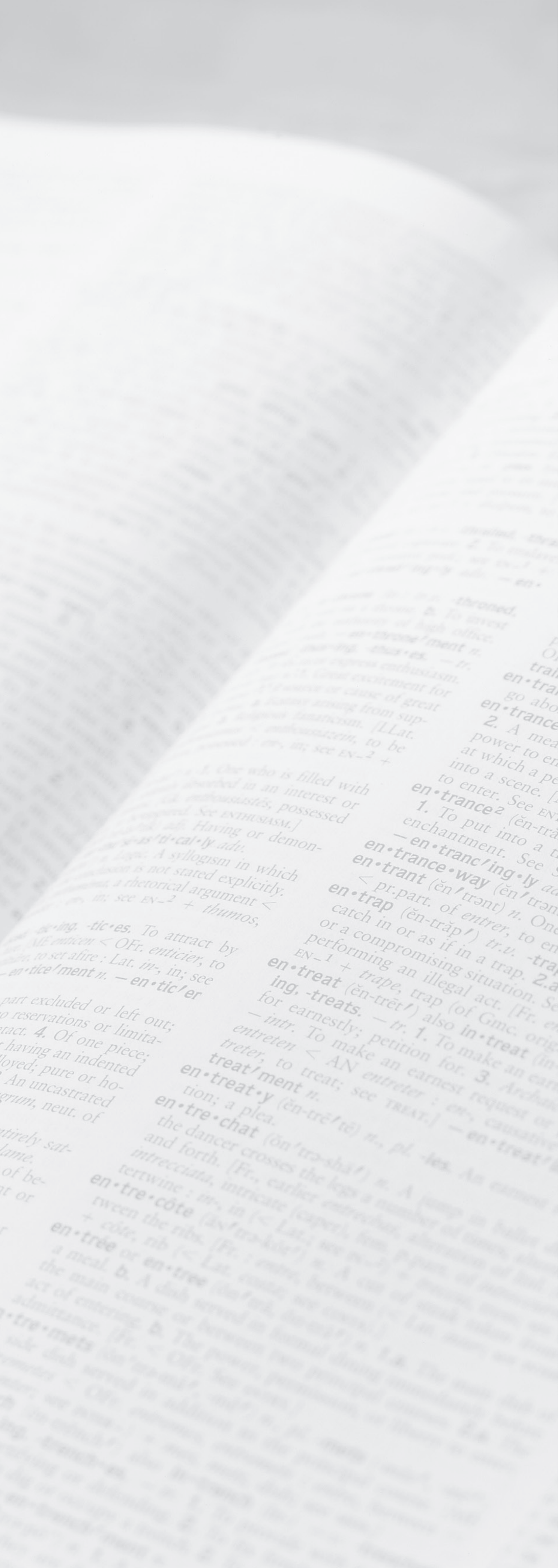

Chapter 29
BC Calculus Practice Test

AP® Calculus BC Exam

SECTION I: Multiple-Choice Questions

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour and 45 minutes
Number of Questions
45
Percent of Total Grade
50%
Writing Instrument
Pencil required

Instructions

Section I of this examination contains 45 multiple-choice questions. Fill in only the ovals for numbers 1 through 45 on your answer sheet.

CALCULATORS MAY NOT BE USED IN THIS PART OF THE EXAMINATION.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding oval on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question	Sample Answer
Chicago is a (A) state (B) city (C) country (D) continent (E) village	Ⓐ ● Ⓒ Ⓓ Ⓔ

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all the multiple-choice questions.

About Guessing

Many candidates wonder whether or not to guess the answers to questions about which they are not certain. Multiple choice scores are based on the number of questions answered correctly. Points are not deducted for incorrect answers, and no points are awarded for unanswered questions. Because points are not deducted for incorrect answers, you are encouraged to answer all multiple-choice questions. On any questions you do not know the answer to, you should eliminate as many choices as you can, and then select the best answer among the remaining choices.

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CALCULUS BC

SECTION I, Part A

Time—55 Minutes

Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. Which of the following is a y-coordinate for the equation $y=\frac{1}{2}x^4+\frac{2}{3}x^3-2x^2+6$ when the tangents to the curve equal zero?

(A) $-\frac{35}{6}$

(B) -6

(C) 0

(D) $\frac{34}{3}$

(E) 36

---

2. What is the sum of the series $\sqrt{5}-\frac{5}{2}+\frac{5\sqrt{5}}{3}-\frac{25}{4}+\ldots+(-1)^n\frac{\sqrt{5}^{n+1}}{n+1}+\ldots$?

(A) $\ln\left(1+\sqrt{5}\right)$

(B) $e^{\sqrt{5}}$

(C) $\ln(\sqrt{5})$

(D) $\sqrt{5}$

(E) The series diverges.

GO ON TO THE NEXT PAGE.

3. $\lim_{x \to 0} \dfrac{\sqrt{x+2}+2x-4}{x^3}$

(A) 0

(B) $\dfrac{3\sqrt{2}}{64}$

(C) $\dfrac{\sqrt{2}}{24}$

(D) $\dfrac{\sqrt{2}}{18}$

(E) Undefined

---

4. Find $\dfrac{d^2y}{dx^2}$ at $x = 1$ for $y^2 - y = 2x^3 - 3x^2 - 4x + 7$.

(A) $-\dfrac{26}{9}$

(B) $-\dfrac{22}{27}$

(C) $-\dfrac{22}{25}$

(D) $-\dfrac{10}{9}$

(E) $\dfrac{26}{9}$

GO ON TO THE NEXT PAGE.

5. $\lim_{h \to 0} \frac{\left(2x^2 + 4xh + 2h^2\right) - 2x^2}{h} =$

(A) $2x^2$
(B) $-2x^2$
(C) $4x$
(D) 4
(E) Undefined

---

6. $\int \frac{dx}{4x^2 - 20x + 26} =$

(A) $\tan^{-1}\left(2x - 5\right) + C$
(B) $\sin^{-1}\left(x - 5\right) + C$
(C) $\tan^{-1}\left(x - 5\right) + C$
(D) $\frac{1}{2}\tan^{-1}\left(2x - 5\right) + C$
(E) $\frac{1}{2}\sin^{-1}\left(2x - 5\right) + C$

---

7. $\int \frac{14x - 12}{(x^2 + 9)\left(x + 3\right)} dx =$

(A) $\left(\frac{3}{2}\right)\ln\left|x^2 + 9\right| + \left(\frac{5}{3}\right)\tan^{-1}\frac{x}{3} + 3\ln\left|x + 3\right| + C$
(B) $\left(\frac{3}{2}\right)\ln\left|x^2 + 9\right| + \left(\frac{5}{3}\right)\tan^{-1}\frac{x}{3} - 3\ln\left|x + 3\right| + C$
(C) $\left(\frac{3}{2}\right)\ln\left|x^2 + 9\right| - \left(\frac{5}{3}\right)\tan^{-1}\frac{x}{3} - 3\ln\left|x + 3\right| + C$
(D) $\left(\frac{3}{2}\right)\ln\left|x^2 + 9\right| - \left(\frac{5}{3}\right)\tan^{-1}\frac{x}{3} + 3\ln\left|x + 3\right| + C$
(E) $-\left(\frac{3}{2}\right)\ln\left|x^2 + 9\right| + \left(\frac{5}{3}\right)\tan^{-1}\frac{x}{3} - 3\ln\left|x + 3\right| + C$

GO ON TO THE NEXT PAGE.

8. If $\frac{dy}{dx} = 2x^3 y$ and $y(0) = 4$, find an equation for y in terms of x.

(A) $y = e^{2x^4}$

(B) $y = 4e^{2x^4}$

(C) $y = 4e^{x^4}$

(D) $y = e^{\frac{x^4}{2}}$

(E) $y = 4e^{\frac{x^4}{2}}$

---

9. Find the derivative of $y^3 = (x+2)^2(2x-3)^3$

(A) $\frac{y}{3}\left(\frac{2}{x+2} + \frac{3}{2x-3}\right)$

(B) $\frac{y}{3}\left(\frac{2}{x+2} + \frac{6}{2x-3}\right)$

(C) $\frac{3}{y}\left(\frac{2}{x+2} + \frac{3}{2x-3}\right)$

(D) $\frac{3}{y}\left(\frac{2}{x+2} + \frac{6}{2x-3}\right)$

(E) $\frac{y}{3}\left(\frac{2}{x+2} - \frac{6}{2x-3}\right)$

GO ON TO THE NEXT PAGE.

10. $\frac{dy}{dx} = (x^3 - 3)y^2$ and $f(2) = \frac{1}{2}$. Find an equation for y in terms of x.

(A) $y = \frac{4}{12x - x^4}$

(B) $y = \frac{4}{x^4 - 12x}$

(C) $y = \frac{1}{3x - x^4} - \frac{1}{2}$

(D) $y = \frac{1}{x^4 - 3x}$

(E) $y = \frac{4}{12x - x^4} + 2$

---

11. Find the derivative of $y = \cos^{-1}(x^2 + 2x)$.

(A) $\frac{-2x - 2}{\sqrt{1 - (x^2 + 2x)^2}}$

(B) $\frac{2x + 2}{\sqrt{1 - (x^2 + 2x)^2}}$

(C) $\frac{-1}{\sqrt{1 - (2x + 2)^2}}$

(D) $\frac{1}{\sqrt{1 - (2x + 2)^2}}$

(E) $\frac{-1}{\sqrt{1 - (x^2 + 2x)^2}}$

GO ON TO THE NEXT PAGE.

12. $\lim_{x \to 2}\left(x^3 - 5x + 3\right) =$

(A) 1
(B) 3
(C) 8
(D) 10
(E) 18

---

13. $\frac{d}{dx}\left(\csc x \sec x\right) =$

(A) $\sec^2 x - \csc^2 x$
(B) $\sec x - \csc x$
(C) $\csc^2 x - \sec^2 x$
(D) $\sec^2 x + \csc^2 x$
(E) $\csc x + \sec x$

---

14. $\frac{d}{dx}\left(\tan\left(\frac{x^3}{x+1}\right)\right) =$

(A) $\frac{3x^3 + 2x^2}{\left(x+1\right)^2}\sec^2\left(\frac{x^3}{x+1}\right)$

(B) $\frac{2x^3 + 3x^2}{\left(x+1\right)^2}\sec^2\left(\frac{x^3}{x+1}\right)$

(C) $\frac{2x^3 - 3x^2}{x+1}\sec^2\left(\frac{x^3}{x+1}\right)$

(D) $\frac{2x^3 - 3x}{\left(x+1\right)^2}\sec^2\left(\frac{x^3}{x+1}\right)$

(E) $\frac{2x^3 + 3x^2}{x+1}\sec^2\left(\frac{x^3}{x+1}\right)$

GO ON TO THE NEXT PAGE.

15. $\lim_{x\to\infty}\dfrac{2x^3+4x^2-6x+7}{12x^3+2x^2+4x-9}=$

(A) 0
(B) $\dfrac{1}{6}$
(C) $\dfrac{1}{3}$
(D) $\dfrac{1}{2}$
(E) The limit is undefined.

---

16. Where is the tangent line perpendicular to the y-axis for the curve $y=2x^4-4x^2+7$ located?

(A) $y=5$
(B) $y=-7$
(C) $x=5$
(D) $y=1$
(E) $x=7$

---

17. If f is continuous on the interval [–3,3] and differentiable everywhere on (–3,3), find $x = c$, where $f(c)$ is the mean value of $f(x)=x^3-3x^2+x-4$.

(A) –2
(B) –1
(C) 0
(D) 1
(E) 2

GO ON TO THE NEXT PAGE.

18. A toy manufacturer has determined the total profit for a month can be determined by the equation $P = -3x^2 + 30x + 150$, where x is the number of thousands of toys sold. How many thousands of toys should be sold to maximize the profit that month?

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

---

19. Find the derivative of $f(x) = x^{x^2}$.

(A) $x^{\left(2x^2\right)}(1 + 2\ln x)$

(B) $e^{x^2 \ln x}(1 + 2\ \ln x)$

(C) $x^{x^2}(x + 2x\ln x)$

(D) $x^{x^2}(x^2 + 2x\ln x)$

(E) $e^{x^2 \ln x}(x + 2x\ln x)$

---

20. If $f(x) = x^2\left(\sqrt[3]{x-4}\right)$, $f'(x) =$

(A) $y\left(\dfrac{2}{x} + \dfrac{1}{3x-12}\right)$

(B) $\dfrac{2}{x} + \dfrac{1}{3x-12}$

(C) $y\left(\dfrac{2}{x} + \dfrac{1}{x-4}\right)$

(D) $y\left(\dfrac{2}{x} - \dfrac{1}{3(x-4)}\right)$

(E) $\dfrac{2}{x} - \dfrac{1}{x-4}$

GO ON TO THE NEXT PAGE.

21. Find the equation of the line normal to the graph of $y = \dfrac{3x^2 + 6}{x + 1}$ at (2,6).

(A) $y = -\dfrac{1}{2}x + 5$

(B) $y = 2x - 7$

(C) $y = \dfrac{1}{2}x + 5$

(D) $y = -\dfrac{1}{2}x + 7$

(E) $y = -2x + 2$

---

22. Find the value of *C* that satisfies the MVTD for $f(x) = 2x^{\frac{3}{2}} + 5x - 2$ on the interval [0,4].

(A) $\dfrac{16}{9}$

(B) 1

(C) $\dfrac{4}{3}$

(D) 9

(E) 8

GO ON TO THE NEXT PAGE.

23. If $y = 7^{2x\cos^2 x}$, then $\frac{dy}{dx} =$

(A) $7^{2x\cos^2 x}\left(2\cos^2 x + 4x\cos x\sin x\right)$

(B) $7^{2x\cos^2 x}\left(2\cos^2 x - 4x\cos x\sin x\right)$

(C) $7^{2x\cos^2 x}\left(\ln 7\right)\left(2\cos^2 x + 4x\cos x\sin x\right)$

(D) $49^{x\cos^2 x}\left(\ln 7\right)\left(2\cos^2 x + 4x\cos x\sin x\right)$

(E) $49^{x\cos^2 x}\left(\ln 7\right)\left(2\cos^2 x - 4x\cos x\sin x\right)$

---

24. Find the derivative of the inverse of $y = x^3 - 3x + 5$ when $y = 7$.

(A) $\frac{1}{9}$

(B) 0

(C) 9

(D) 144

(E) $\frac{1}{144}$

---

25. Find the derivative of the inverse of $f\left(x\right) = \sin^2\left(6\pi - x\right)$ at $f\left(x\right) = \frac{1}{2}$ for $0 \le x \le \frac{\pi}{2}$.

(A) -1

(B) $-\frac{\sqrt{2}}{2}$

(C) $\frac{1}{2}$

(D) $\frac{\sqrt{2}}{2}$

(E) 1

GO ON TO THE NEXT PAGE.

26. $\int\left(3x^3 - 2x^2 + x - 7\right)dx =$

(A) $12x^4 - 6x^3 + 2x^2 + 7x + C$

(B) $x^2 - x + C$

(C) $12x^4 - 6x^3 + 2x^2 - 7x + C$

(D) $\frac{3}{4}x^4 - \frac{2}{3}x^3 + \frac{x^2}{2} - 7x + C$

(E) $\frac{3}{4}x^4 + \frac{2}{3}x^3 - \frac{x^2}{2} + 7x + C$

---

27. Find the derivative of $y = \sqrt{\frac{1+x}{3x-1}}\left(\frac{x+2}{x-1}\right)^2$?

(A) $\frac{y}{2}\left(\frac{1}{x+1} - \frac{1}{3x+1}\right) + 2y\left(\frac{1}{x+2} - \frac{1}{x-1}\right)$

(B) $\frac{y}{2}\left(\frac{1}{1+x} - \frac{3}{3x-1}\right) + \frac{1}{x+2} - \frac{1}{x-1}$

(C) $2y\left(\frac{1}{x+1} - \frac{1}{3x+1}\right) + \frac{y}{2}\left(\frac{1}{x+2} - \frac{1}{x-1}\right)$

(D) $\frac{y}{2}\left(\frac{1}{x+1} - \frac{3}{3x-1}\right) + 2y\left(\frac{1}{x+2} - \frac{1}{x-1}\right)$

(E) $2y\left(\frac{1}{1+x} - \frac{3}{3x-1}\right) + \frac{y}{2}\left(\frac{1}{x+2} - \frac{1}{x-1}\right)$

GO ON TO THE NEXT PAGE.

28. Find $\lim_{x \to \frac{\pi}{2}} \frac{3x}{2\cos x}$.

(A) $\frac{3}{2}$

(B) $-\frac{3}{2}$

(C) $\frac{2}{3}$

(D) 3

(E) 0

END OF PART A, SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS BC

SECTION I, Part B

Time—50 Minutes

Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

1. The **exact** numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

2. Unless otherwise specified, the domain of a function *f* is assumed to be the set of all real numbers *x* for which *f*(*x*) is a real number.

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29. $\int x^2 e^{4x^3+7} dx =$

(A) $12e^{4x^3+7} + C$

(B) $e^{4x^3+7} + C$

(C) $\frac{1}{12}e^{4x^3+7} + C$

(D) $\frac{1}{2}e^{4x^3+7} + C$

(E) $2e^{4x^3+7} + C$

GO ON TO THE NEXT PAGE.

30. $\displaystyle\int \frac{x+7}{(2x-3)(x+6)}\,dx =$

(A) $\frac{17}{30}\ln|2x-3| - \frac{1}{15}\ln|x+6| + C$

(B) $\frac{1}{15}\ln|2x-3| - \frac{17}{30}\ln|x+6| + C$

(C) $\frac{17}{30}\ln|2x-3| + \frac{1}{15}\ln|x+6| + C$

(D) $\frac{17}{15}\ln|2x-3| - \frac{1}{15}\ln|x+6| + C$

(E) $\frac{17}{15}\ln|2x-3| + \frac{1}{15}\ln|x+6| + C$

---

31. Find b where $y = x^2 - ax + b$ and $y = x^2 + cx$ have a common tangent at (2,1).

(A) $-\frac{3}{2}$

(B) $\frac{3}{2}$

(C) –8

(D) 8

(E) 0

---

32. Car A and car B leave a town at the same time. Car A drives due north at a rate of 60 km/hr and car B goes east at a rate of 80 km/hr. How fast is the distance between them increasing after 2 hours?

(A) 120 km/hr
(B) 160 km/hr
(C) 100 km/hr
(D) 200 km/hr
(E) 70 km/hr

GO ON TO THE NEXT PAGE.

33. Approximate cos 91°.

(A) $-\frac{179\pi}{180}$

(B) $\frac{179\pi}{180}$

(C) $\frac{\pi}{180}$

(D) $-\frac{\pi}{180}$

(E) 0

---

34. A 30-foot ladder leaning against a wall is pushed up the wall at a rate of 3 ft/sec. How fast is the ladder sliding across the ground towards the wall when it is 18 feet up the wall from the ground?

(A) $-\frac{9}{4}$ ft/sec

(B) $-\frac{4}{9}$ ft/sec

(C) $-\frac{3}{2}$ ft/sec

(D) -4 ft/sec

(E) -3 ft/sec

---

35. Approximate the area under the curve $y = x^2 + 2$ from $x = 1$ to $x = 2$ using four left-endpoint rectangles.

(A) 4.333
(B) 3.969
(C) 4.719
(D) 4.344
(E) 4.328

GO ON TO THE NEXT PAGE.

36. What is the mean value of $f(x) = \dfrac{\cos x + 1}{2x^2}$ over the interval (–1,1)?

(A) –196
(B) –1
(C) 1
(D) 196
(E) There is no such value.

---

37. Find the second derivative of $y = x^5 - 2x^2 + 7$ at $x = 1$.

(A) 0
(B) 1
(C) 12
(D) 16
(E) 20

---

38. What is $\dfrac{dy}{dx}$ if $y = 3^{x^2}$?

(A) $2x\ln 3\left(3^{x^2}\right)$

(B) $2x\ln 3$

(C) 3^{x^2}

(D) $2x3^{x^2}$

(E) $\ln 3\left(3^{x^2}\right)$

GO ON TO THE NEXT PAGE.

39. $\int_{\ln 2}^{2} \frac{x^3 + x^2 - 2x}{x^2 + x - 2} dx$

(A) $\frac{(\ln 2)^2 - 4}{2}$

(B) $\frac{4 - (\ln 2)^2}{2}$

(C) $\frac{\ln 4 - 4}{2}$

(D) $\frac{4 - \ln 4}{2}$

(E) $2 - (\ln 2)^2$

---

40. Find the derivative of the inverse of $y = 2x^2 - 8x + 9$ at $y = 3$.

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) 1

(D) 3

(E) 4

GO ON TO THE NEXT PAGE.

41. A frame is bought for a photo that is 144 in^2. The artist would like to have a mat that is 3 in on the top and bottom and 6 in on each side. Find the dimensions of the frame that will minimize its area.

(A) $\left(12\sqrt{2}+12\right)\text{ in}\times\left(6\sqrt{2}+6\right)\text{ in}$

(B) $\left(12\sqrt{2}+6\right)\text{ in}\times\left(6\sqrt{2}+12\right)\text{ in}$

(C) $\left(12\sqrt{2}\right)\text{ in}\times\left(6\sqrt{2}\right)\text{ in}$

(D) $\left(12\sqrt{2}+6\right)\text{ in}\times\left(6\sqrt{2}+3\right)\text{ in}$

(E) $\left(12\sqrt{2}+3\right)\text{ in}\times\left(6\sqrt{2}+6\right)\text{ in}$

---

42. What is the length of the curve $y=\frac{2}{3}x^{\frac{3}{2}}$ from $x = 1$ to $x = 6$.

(A) $\frac{4}{3}\sqrt{7}-\frac{14}{3}\sqrt{2}$

(B) $\frac{14}{3}\sqrt{7}+\frac{4}{3}\sqrt{2}$

(C) $\frac{14}{3}\sqrt{7}-\frac{4}{3}\sqrt{2}$

(D) $\frac{4}{3}\sqrt{2}-\frac{14}{3}\sqrt{7}$

(E) $\frac{14}{3}\sqrt{2}+\frac{4}{3}\sqrt{7}$

---

43. Find the length of the curve defined by $x=\left(\frac{1}{2}\right)t^2+7$ and $y=\left(\frac{8}{3}\right)(t+4)^{\frac{3}{2}}$ from $t = 0$ to $t = 8$.

(A) 36
(B) 64
(C) 80
(D) 96
(E) 100

GO ON TO THE NEXT PAGE.

44. Use Euler's method with $h = 0.2$ to estimate $y = 1$ if $y' = \dfrac{y^2 - 1}{2}$ and $y(0) = 0$.

(A) 7.690
(B) 12.730
(C) 13.504
(D) 29.069
(E) 90.676

---

45. Which of the following series converges?

(A) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n$

(B) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}$

(C) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n-1}$

(D) $\displaystyle\sum_{n=1}^{\infty}\frac{4^n}{n^2}$

(E) $\displaystyle\sum_{n=1}^{\infty}3^n$

STOP

END OF PART B, SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART B ONLY.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

SECTION II
GENERAL INSTRUCTIONS

You may wish to look over the problems before starting to work on them, since it is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

- You should write all work for each part of each problem in the space provided for that part in the booklet. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. You will be graded on the correctness and completeness of your methods as well as your answers. Correct answers without supporting work may not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons and that you clearly identify functions, graphs, tables, or other objects you use.
- You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt (X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

GO ON TO THE NEXT PAGE.

SECTION II, PART A
Time—30 minutes
Number of problems—2

A graphing calculator is required for some problems or parts of problems.

During the timed portion for Part A, you may work only on the problems in Part A.

On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

1.

x	2	2.2	2.4	2.6	2.8	3
$\frac{dy}{dx}$	6	5	4	2.5	1	0.5

The equation for y is thrice differentiable for $x > 0$ with $y = 3$ at $x = 2$, the second derivative is equal to 2 at $x = 2$, and the third derivative is 4 at $x = 2$. Values of the first derivative are given for select values of x above.

(a) Write an equation for the tangent line of y at $x = 3$. Use this line to approximate y at $x = 3$.

(b) Use a right endpoint Riemann sum with five subintervals of equal length and values from the table to approximate $\int_2^3 \frac{dy}{dx}\,dx$. Use this approximation to estimate y at $x = 3$. Show your work.

(c) Use Euler's Method, starting at $x = 2$ with five steps of equal size, to approximate y at $x = 3$. Show your work.

(d) Write a third degree Taylor polynomial for y about $x = 2$. Use it to approximate y at $x = 3$.

---

2. Let R be the region bound by $y_1 = 2x^3 - 4x^2 - 8$ and $y_2 = 2x^2 + 8x - 8$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis for $0 \le x \le 4$.

(c) Find the volume of the solid generated when R is revolved about the line $x = 2$ when $-1 \le x \le 0$.

GO ON TO THE NEXT PAGE.

SECTION II, PART B
Time—1 hour
Number of problems—4

No calculator is allowed for these problems.

During the timed portion for Part B, you may continue to work on the problems in Part A without the use of any calculator.

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3. Consider the graph of the polar curve $r = 1 + 2\sin\theta$ for $0 \le \theta \le 2\pi$. Let S be the region bound between the inner and outer loops.

   (a) Write an integral expression for the area of S.

   (b) Write expressions for $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$ in terms of θ.

   (c) Write an equation in terms of x and y for the line normal to the graph of the polar curve at the point where $\theta = \frac{3\pi}{2}$. Show your work.

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4. For $t \ge 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 3$, the particle is at position (3,1). It is known that $\frac{dx}{dt} = e^{-2t}(t+1)^2$ and $\frac{dy}{dt} = \cos^2 t$.

   (a) Is the horizontal movement to the left or to the right at time $t = 3$? Find the slope of the particle's path at $t = 3$.
   (b) Find the y-coordinate of the particle's position at time $t = 6$.
   (c) Find the speed and acceleration of the particle at $t = 6$.
   (d) Find the distance traveled by the particle from time $t = 3$ to $t = 6$.

GO ON TO THE NEXT PAGE.

5. A particle's position in the *xy*-plane at any time t is given by $x = 3t^3 - 4$ and $y = 2t^5 - 3t^3$. Find:

(a) The x and y components of the particle's velocity.

(b) $\dfrac{dy}{dx}$ at $t = 3$.

(c) The acceleration of the particle at $t = 3$.

(d) The time(s) when the particle is changing direction.

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6. Let $y_1' = \dfrac{3x^2}{y_1^2}$ and $y_2' = 2x^3y_2 - xy_2$.

(a) If $x = 0$ and $y = 6$, find y_1.

(b) If $x = -2$ and $y = e^2$, find y_2.

(c) Use Euler's method to approximate y_1 when $x = 3$. Start at $x = 0$ using three steps. Check your answer against the real value of y_1 at $x = 3$. Is this a reasonable approximation?

STOP

END OF EXAM