

Chapter 23

Areas, Volumes, and Average Values Drill

AREAS, VOLUMES, AND AVERAGE VALUES DRILL

1. What is the area between the curve $y = x^3 - 8$ and the x -axis from $x = 0$ to $x = 2$.

(A) 0
(B) 4
(C) 8
(D) 12
(E) 16

2. What is the area enclosed by the curve $x = y^2 - y - 2$ and the y -axis?

(A) 3
(B) 3.5
(C) 4
(D) 4.5
(E) 5

3. What is the volume of the solid formed by revolving the curve $y = 2x^2 - 8$ about the x -axis?

(A) 21.333
(B) 67.021
(C) 136.533
(D) 221.867
(E) 428.932

4. What is the volume of the solid formed by the curves

$$y = \frac{3}{2}x^2 \text{ and } y = 3x \text{ revolved around the line } y = 7?$$

(A) $\frac{87}{4}\pi$
(B) $\frac{92}{4}\pi$
(C) $\frac{87}{5}\pi$
(D) $\frac{92}{5}\pi$
(E) $\frac{97}{5}\pi$

5. Find the area of the region in the plane enclosed by the cardioid $r = 2 + 2\sin\theta$.

(A) π
(B) 2π
(C) 3π
(D) 6π
(E) 12π

6. Find the volume of the solid formed by revolving $y = x^2$ from $x = 1$ to $x = 3$ over the x -axis.

(A) 20π
(B) $\frac{124}{5}\pi$
(C) $\frac{240}{5}\pi$
(D) $\frac{242}{5}\pi$
(E) 50π

7. Find the area between the curves $y = x^4$ and $y = x^2$ from $x = 0$ to $x = 1$.

- (A) $\frac{1}{15}$
- (B) $\frac{2}{15}$
- (C) $\frac{1}{5}$
- (D) $\frac{1}{3}$
- (E) 1

8. Approximate the area under the curve $f(x) = x^3 + 4$ from $x = 0$ to $x = 2$ using four inscribed trapezoids.

- (A) $\frac{33}{4}$
- (B) 10
- (C) $\frac{59}{4}$
- (D) 12
- (E) $\frac{49}{4}$

9. Given the following table of values for x and y :

x	0	1	3	4	5	7	10	13	15
$f(x)$	2	7	10	9	6	8	12	15	20

Use a left-hand Riemann sum with eight subintervals to approximate $\int_0^{15} f(x) dx$.

- (A) 110
- (B) 121
- (C) 123
- (D) 126
- (E) 137

10. What is the area between the curves $y = 6x^2 - x$ and $y = x^2 - 6x$?

- (A) 4
- (B) $\frac{25}{6}$
- (C) $\frac{9}{2}$
- (D) $\frac{14}{3}$
- (E) $\frac{29}{6}$

11. Which of the following would yield the area between the curves $y = x^2$ and $y = 2x - x^2$?

- (A) $\int_0^1 (2x^2 + 2x) dx$
- (B) $\int_0^1 (2x - 2x^2) dx$
- (C) $\int_0^1 (2x^2 - 2x) dx$
- (D) $\int_0^1 (x^2 - 2x) dx$
- (E) $\int_0^1 (x - x^2) dx$

12. Which of the following would yield the area between the equations $y = 5x - x^2$ and $y = x$?

- (A) $\int_0^1 (4x - x^2) dx$
- (B) $\int_1^4 (4x - x^2) dx$
- (C) $\int_0^2 (4x - x^2) dx$
- (D) $\int_2^4 (4x - x^2) dx$
- (E) $\int_0^4 (4x - x^2) dx$

13. Which of the following would yield the area between $y = x$ and $y = x^2$?
- (A) $\int_0^4 (x - x^2) dx$
 (B) $\int_0^1 (x^2 - x) dx$
 (C) $\int_0^1 (x - x^2) dx$
 (D) $\int_0^4 (x^2 - x) dx$
 (E) $\int_0^1 (x - 2x^2) dx$
14. Find the volume of the solid obtained by rotating about the x -axis the region under the curve $y = \sqrt{x}$ from 0 to 1.
- (A) $\pi \int_0^1 x^3 dx$
 (B) $\pi \int_0^1 x dx$
 (C) $\pi \int_0^1 x^2 dx$
 (D) $2\pi \int_0^1 x^2 dx$
 (E) $2\pi \int_0^1 x dx$
15. Which of the following would calculate the volume of the solid obtained by rotating the region bounded by $y = x^3$, $y = 8$, and $x = 0$ about the y -axis?
- (A) $\pi \int_0^8 x^6 dx$
 (B) $2\pi \int_0^8 y^{\frac{1}{3}} dy$
 (C) $\pi \int_0^8 y^{\frac{2}{3}} dy$
 (D) $2\pi \int_0^8 y^{\frac{4}{3}} dy$
 (E) $2\pi \int_0^8 x^{\frac{2}{3}} dx$
16. What is the average value of the function $y = 4x - x^2$ on the interval $[0,4]$?
- (A) 6
 (B) $\frac{19}{3}$
 (C) $\frac{20}{3}$
 (D) 7
 (E) $\frac{22}{3}$
17. Find the average value of $f(x) = 2\sin x - \sin 2x$ from 0 to π .
- (A) $\frac{4}{\pi}$
 (B) $\frac{3}{\pi}$
 (C) $\frac{2}{\pi}$
 (D) $\frac{1}{\pi}$
 (E) $-\frac{1}{\pi}$
18. Find the average value of $f(x) = x\sqrt{1+x^2}$ from 0 to 5.
- (A) $\frac{2}{3} \int_0^5 [x\sqrt{1+x^2}] dx$
 (B) $\frac{1}{3} \int_0^5 [x\sqrt{1+x^2}] dx$
 (C) $\frac{1}{6} \int_0^5 [x\sqrt{1+x^2}] dx$
 (D) $\frac{1}{5} \int_0^5 [x\sqrt{1+x^2}] dx$
 (E) $\frac{1}{10} \int_0^5 [x\sqrt{1+x^2}] dx$