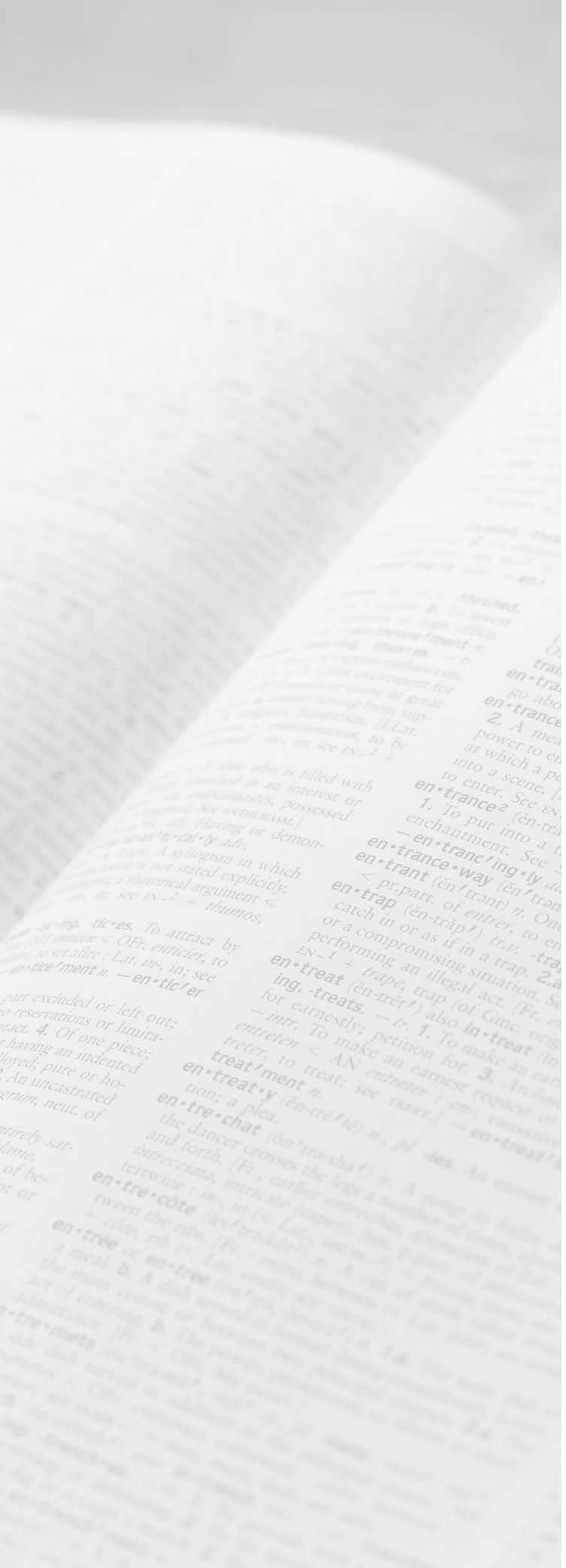

Chapter 15
Applications of Derivatives Drill 2

APPLICATIONS OF DERIVATIVES DRILL 2

1. What is the equation of a parabola $y = ax^2 + bx + c$ that passes through (1, 4) and whose tangent lines at $x = -1$ and $x = 5$ have slopes 6 and –2, respectively?

   (A) $y = \frac{2}{3}x^2 + \frac{14}{3}x$

   (B) $y = \frac{14}{3}x^2 + \frac{2}{3}x$

   (C) $y = \frac{2}{3}x^2 - \frac{14}{3}x$

   (D) $y = -\frac{14}{3}x^2 - \frac{2}{3}x$

   (E) $y = -\frac{2}{3}x^2 + \frac{14}{3}x$

2. At what point(s) on the curve $y = \sin x + \cos x$, $0 \leq x \leq 2\pi$, is the tangent line horizontal?

   (A) $\left(\frac{\pi}{4}, \sqrt{2}\right)$

   (B) $\left(\frac{\pi}{4}, \sqrt{2}\right)$ and $\left(\frac{5\pi}{4}, \sqrt{2}\right)$

   (C) $\left(\frac{5\pi}{4}, -\sqrt{2}\right)$

   (D) $\left(\frac{5\pi}{4}, \sqrt{2}\right)$

   (E) $\left(\frac{\pi}{4}, -\sqrt{2}\right)$ and $\left(\frac{5\pi}{4}, \sqrt{2}\right)$

3. The volume of a cube is increasing at a rate of 10 cm^3/min. How fast is the surface area increasing when the length of an edge is 30 cm?

   (A) $\frac{4}{3}$ cm^2/min

   (B) $\frac{3}{4}$ cm^2/min

   (C) $\frac{3}{2}$ cm^2/min

   (D) $\frac{2}{3}$ cm^2/min

   (E) $\frac{5}{4}$ cm^2/min

4. How long does it take for a ball to reach 35 m/s if it is pushed down a hill and its position at time t, in seconds, is given by $s = 5t + 3t^2$, in meters?

   (A) 2 seconds
   (B) 3 seconds
   (C) 4 seconds
   (D) 5 seconds
   (E) 6 seconds

5. What is the maximum height reached by a ball if it travels according to the function $s = 80t - 16t^2$, in meters?

   (A) 100
   (B) 80
   (C) 60
   (D) 50
   (E) 40

6. What is an equation of the line tangent to $y^2 = x^3 + 3x^2$ at the point (1,–2)?

   (A) $4x + 9y = 1$
   (B) $9x - 4y = 1$
   (C) $4x - 9y = 1$
   (D) $4y - 9x = 1$
   (E) $9x + 4y = 1$

7. Find the point on the curve $y = x^{\frac{1}{2}}$ that is a minimum distance from the point (16,0).

(A) $\left(\frac{7}{2}, \sqrt{\frac{7}{2}}\right)$

(B) $(16,4)$

(C) $\left(\frac{31}{2}, \sqrt{\frac{31}{2}}\right)$

(D) $\left(\frac{33}{2}, \sqrt{\frac{33}{2}}\right)$

(E) $\left(2, \sqrt{2}\right)$

8. A cone-shaped funnel has a diameter of 10 m and a height of 12 m. Find the error in the volume if the height is exact, but the diameter is 10.2 m.

(A) $4\pi\, m^3$

(B) $12\pi\, m^3$

(C) $\frac{5}{3}\pi\, m^3$

(D) $20\pi\, m^3$

(E) $\frac{5}{6}\pi\, m^3$

9. Find the length of the curve $x = t^2 + 3$ and $y = 2t^2 - 7$ from $t = 2$ to $t = 5$.

(A) 156
(B) 78
(C) $75\sqrt{5}$
(D) $39\sqrt{5}$
(E) $21\sqrt{5}$

10. Find a point on the curve $y = x^3 - 4x^2 - 3x + 13$ where the normal is parallel to the y-axis.

(A) $(3,-5)$

(B) $\left(-\frac{1}{3}, -\frac{365}{27}\right)$

(C) $(0,0)$

(D) $(3,5)$

(E) $\left(\frac{1}{3}, \frac{365}{27}\right)$

11. Find the slope of the tangent line to the curve $r = 2 + 3\sin\theta$.

(A) $\frac{3\sin 2\theta + 2\cos\theta}{3\cos^2\theta - 2\sin\theta - 3\sin\theta\cos\theta}$

(B) $\frac{3\sin 2\theta + 2\cos\theta}{3\cos^2\theta - 2\sin\theta + 3\sin\theta\cos\theta}$

(C) $\frac{3\sin\theta\cos\theta + 2\cos\theta}{3\sin^2\theta - 2\sin\theta - 3\sin\theta\cos\theta}$

(D) $\frac{3\sin\theta\cos\theta - 2\cos\theta}{3\sin^2\theta - 2\sin\theta - 3\sin\theta\cos\theta}$

(E) $\frac{3\sin 2\theta + 2\cos\theta}{3\cos^2\theta + 2\sin\theta + 3\sin\theta\cos\theta}$

12. Find the equation of the line tangent to the curve $3x^3 - 2x^2 + x = y^3 + 2y^2 + 3y$ at $y = -2$.

(A) $y = 2x - 1$
(B) $y = 2x$
(C) $y = \frac{45}{2}x$
(D) $y = \frac{6}{23}x$
(E) $y = 2x - 2$

13. The curve $y = ax^3 + bx^2 + cx + d$ passes through the point (2, 8) and is normal to $y = -\frac{1}{3}x - 4$ at (0, –4). If $b = 5$, what is the value of a?

(A) –2

(B) $-\frac{7}{4}$

(C) $-\frac{3}{2}$

(D) –1

(E) $-\frac{1}{2}$

14. At what time does the particle change direction if the position function is given by $x(t) = 2x^4 - 4x^3 + 2x^2 - 8$, where $t > 0$?

(A) $\frac{1}{4}$

(B) 1

(C) $\frac{3}{2}$

(D) 2

(E) $\frac{5}{2}$

15. What is the particle's velocity $\left(\frac{dy}{dx}\right)$ at $t = 3$ if $x = 3x^3 - 2x^2 + 4$ and $y = 2x^2 + 3x - 7$?

(A) 69

(B) 15

(C) $\frac{5}{23}$

(D) 4

(E) $\frac{23}{5}$

16. Use differentials to approximate $(5.2)^3$.

(A) 125
(B) 130
(C) 135
(D) 140
(E) 145

17. The radius of a cylinder is increased from 9 to 9.03 inches. If the height remains constant at 12 inches. Estimate the change in volume.

(A) 0.005π in^3
(B) 3.24π in^3
(C) 6.48π in^3
(D) 9π in^3
(E) 12π in^3

18. Use differentials to approximate cos275°.

(A) $\frac{\pi}{72}$

(B) $\frac{\pi}{36}$

(C) $\frac{\pi}{18}$

(D) $\frac{\pi}{9}$

(E) $\frac{\pi}{3}$

19. Find the length of the curve $y = \frac{4}{3}x^{\frac{3}{2}}$ from $x = 0$ to $x = 6$.

(A) $\frac{59}{3}$

(B) 20

(C) $\frac{61}{3}$

(D) $\frac{62}{3}$

(E) 21

20. If $x^2 + y^2 = 25$, then find the slope of the tangent that passes through the point(2,4).

(A) -1

(B) $-\frac{1}{2}$

(C) 0

(D) $\frac{1}{2}$

(E) 1

21. Find the equation of the tangent line to the curve $x^2 + xy + y^2 = 3$ at (1,1).

(A) $3x - y = 2$
(B) $x - y = 2$
(C) $3x + y = 2$
(D) $x + y = 2$
(E) $x - 3y = -2$

22. What is the slope of the line normal to the curve $f(x) = x^4 + 3x^2$ that passes through the point (2,1)?

(A) 44

(B) $\frac{1}{44}$

(C) 0

(D) $-\frac{1}{44}$

(E) -44

23. What dimensions must a rectangle have to maximize the area and have a perimeter of 100 meters?

(A) 40 m by 10 m
(B) 45 m by 5 m
(C) 35 m by 15 m
(D) 30 m by 20 m
(E) 25 m by 25 m

24. What two positive numbers not only yield a minimum sum, but also produce a product of 100?

(A) 50 and 2
(B) 25 and 4
(C) 10 and 10
(D) 20 and 5
(E) 100 and 1

25. What two numbers both have a sum of 23 and a product that is maximized?

(A) 12 and 11
(B) 11.5 and 11.5
(C) 10 and 13
(D) 9.5 and 13.5
(E) 8 and 15

26. If $y = 4x^3 - 9x^2 + 6x$, then what is the value of the relative minimum, if any?

(A) -1

(B) 0

(C) $\frac{1}{2}$

(D) 1

(E) 2