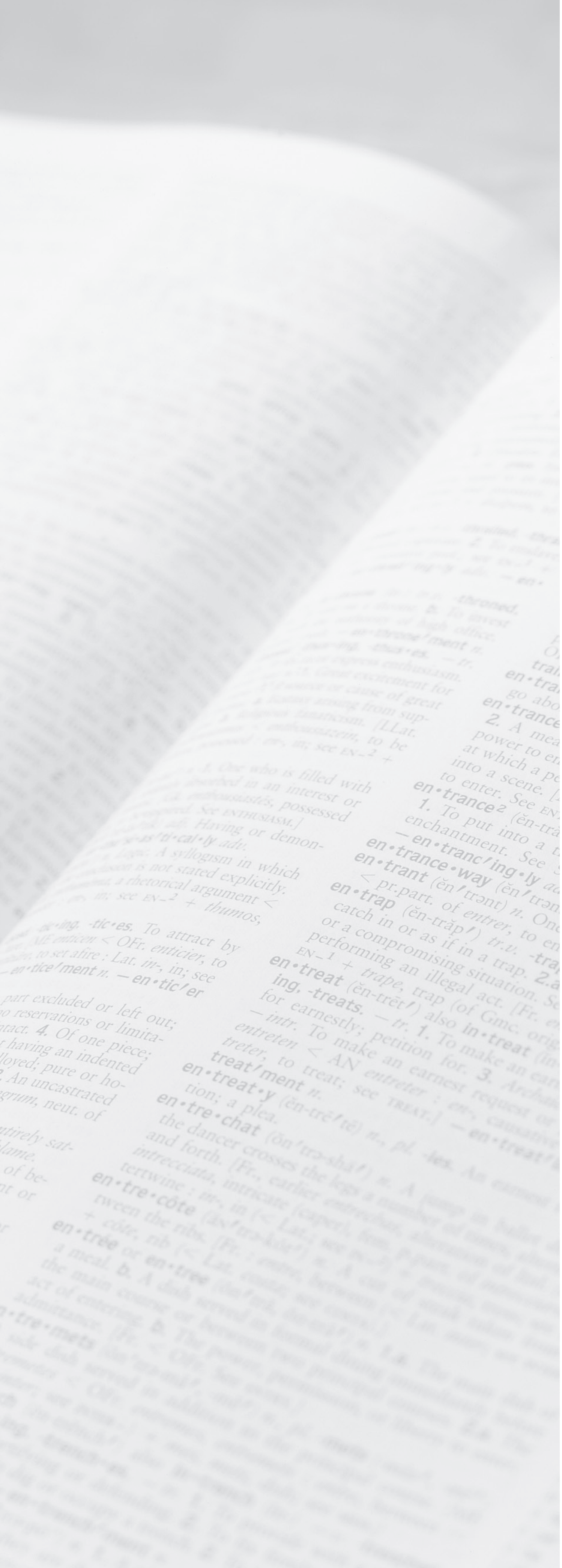

Chapter 27
AB Calculus Practice Test

AP® Calculus AB Exam

SECTION I: Multiple-Choice Questions

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour and 45 minutes
Number of Questions
45
Percent of Total Grade
50%
Writing Instrument
Pencil required

Instructions

Section I of this examination contains 45 multiple-choice questions. Fill in only the ovals for numbers 1 through 45 on your answer sheet.

CALCULATORS MAY NOT BE USED IN THIS PART OF THE EXAMINATION.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding oval on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question	Sample Answer
Chicago is a (A) state (B) city (C) country (D) continent (E) village	Ⓐ ● Ⓒ Ⓓ Ⓔ

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all the multiple-choice questions.

About Guessing

Many candidates wonder whether or not to guess the answers to questions about which they are not certain. Multiple choice scores are based on the number of questions answered correctly. Points are not deducted for incorrect answers, and no points are awarded for unanswered questions. Because points are not deducted for incorrect answers, you are encouraged to answer all multiple-choice questions. On any questions you do not know the answer to, you should eliminate as many choices as you can, and then select the best answer among the remaining choices.

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CALCULUS AB

SECTION I, Part A

Time—55 Minutes

Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: Unless otherwise specified, the domain of a function *f* is assumed to be the set of all real numbers *x* for which *f*(*x*) is a real number.

1. Find the second derivative of $x^2y=2$.

   (A) $\dfrac{6y}{x^2}$

   (B) $\dfrac{x^2}{y}$

   (C) $\dfrac{y}{x^2}$

   (D) $-\dfrac{6y}{x^2}$

   (E) $-\dfrac{x^2}{6y}$

GO ON TO THE NEXT PAGE.

2. If $y = \ln\left(6x^3 - 2x^2\right)$, then $f'(x) =$

(A) $\dfrac{9x+2}{3x^2 - x}$

(B) $\dfrac{9x+2}{3x^2 + x}$

(C) $\dfrac{9x-2}{3x^2 - x}$

(D) $\dfrac{9x+2}{3x^2 + x}$

(E) $\dfrac{18x^2+4x}{6x^3 - 2x^2}$

---

3. Find $\lim\limits_{x \to \infty} 3xe^{-3x}$.

(A) $\dfrac{1}{3}$

(B) 3

(C) –1

(D) 1

(E) 0

---

4. The radius of a sphere is measured to be 5 cm with an error of ± 0.1 cm. Use differentials to approximate the error in the volume.

(A) $\pm\pi$ cm^3

(B) $\pm 100\pi$ cm^3

(C) $\pm 10\pi$ cm^3

(D) $\pm 4\pi$ cm^3

(E) $\pm 40\pi$ cm^3

GO ON TO THE NEXT PAGE.

5. A side of a cube is measured to be 10 cm. Estimate the change in surface area of the cube when the side shrinks to 9.8 cm.

(A) $+2.4\text{ cm}^2$
(B) -2.4 cm^2
(C) -120 cm^2
(D) $+24\text{ cm}^2$
(E) -24 cm^2

---

6. Find the derivative of y, when $y^2 = \left(x^2+2\right)(x+3)^2(2x+7)^{\frac{1}{2}}$ at (1,12)?

(A) $\frac{20}{3}$
(B) 7
(C) $\frac{22}{3}$
(D) $\frac{23}{3}$
(E) 8

---

7. $\int \frac{x^3}{2}\,dx =$

(A) $\frac{x^4}{8}+C$
(B) $\frac{x^4}{2}+C$
(C) $2x^4+C$
(D) $\frac{3}{2}x^2+C$
(E) $8x^4+C$

GO ON TO THE NEXT PAGE.

8. $\int x^2 \sin\left(3x^3+2\right)dx =$

(A) $-9\cos\left(3x^3+2\right)+C$

(B) $-\cos\left(3x^3+2\right)+C$

(C) $\dfrac{-\cos\left(3x^3+2\right)}{9}+C$

(D) $\dfrac{\cos\left(3x^3+2\right)}{9}+C$

(E) $9\cos\left(3x^3+2\right)+C$

---

9. If $f(x)=\begin{cases}2ax^2+bx+6, & x\le -1\\ 3ax^3-2bx^2+4x, & x>-1\end{cases}$ and is differentiable for all real values, then b = ?

(A) –13
(B) 0
(C) 45
(D) 55
(E) 110

---

10. $\dfrac{d}{dx}\left(\dfrac{x^3-4x^2+3x}{x^2+4x-21}\right)=$

(A) $\dfrac{x^2-x}{x+7}$

(B) $\dfrac{x-1}{x-7}$

(C) $\dfrac{x^2-14x+7}{(x-7)^2}$

(D) $\dfrac{2x^2+13x-7}{(x+7)^2}$

(E) $\dfrac{x^2+14x-7}{(x+7)^2}$

GO ON TO THE NEXT PAGE.

11. $\lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 - 2x^2}{h}$

(A) 4
(B) $3x^2$
(C) $2x^2$
(D) $4x$
(E) $6x$

---

12. Find the point on the curve $x^2 + y^2 = 9$ that is a minimum distance from the point (1,2).

(A) $\left(\sqrt{5}, 2\right)$
(B) $\left(-\sqrt{5}, -2\right)$
(C) $\left(\sqrt{5}, -2\right)$
(D) $\left(-\sqrt{5}, 2\right)$
(E) $\left(5, 2\right)$

GO ON TO THE NEXT PAGE.

13. Find $\frac{dy}{dx}$ if $y = \log_3(2x^3 + 4x^2)$.

(A) $\dfrac{6x^2 + 8x}{(x^2 + 2x)\ln 3}$

(B) $\dfrac{3\text{x} + 4}{(2\text{x}^3 + 4\text{x}^2)\ln 3}$

(C) $\dfrac{3x + 4}{(x^2 + 2x)\ln 3}$

(D) $\dfrac{3x + 4}{3\ln(x^2 + 2x)}$

(E) $\dfrac{6x^2 + 8x}{(3x^3 + 2x^2)\ln 3}$

---

14. What curve is represented by $x = 2t^3$ and $y = 4t^9$?

(A) $y = 2x^2$

(B) $y = x^2$

(C) $y = 3x^2$

(D) $y = x^3$

(E) $y = 2x^3$

GO ON TO THE NEXT PAGE.

15. Find $\lim_{x\to 0}\dfrac{2x^3-3\sin x}{x^4}$.

(A) -1

(B) $-\dfrac{1}{2}$

(C) 0

(D) $\dfrac{1}{2}$

(E) 1

---

16. $\int 18x^2\sec^2\left(3x^3\right)dx =$

(A) $2\tan^2\left(3x^3\right)+C$

(B) $2\cot^2\left(3x^3\right)+C$

(C) $\cot\left(3x^3\right)+C$

(D) $\tan\left(3x^3\right)+C$

(E) $2\tan\left(3x^3\right)+C$

---

17. What is the equation of the line normal to the curve $y=x^3+2x^2-5x+7$ at $x=1$?

(A) $y=-\dfrac{1}{2}x+\dfrac{11}{2}$

(B) $y=2x+3$

(C) $y=-\dfrac{1}{2}x-\dfrac{11}{2}$

(D) $y=-2x+3$

(E) $y=-2x-\dfrac{11}{2}$

GO ON TO THE NEXT PAGE.

18. Find the value of c that satisfies Rolle's Theorem for $f(x)=\dfrac{x^2+4x-12}{x^2+2x-3}$ on the interval [–6,2].

(A) –6
(B) –3
(C) 1
(D) 2
(E) No such value exists.

---

19. If $\cos^2 x+\sin^2 y=y$, then $\dfrac{dy}{dx}$.

(A) $\dfrac{2\cos x\sin x}{2\cos y\sin y+1}$
(B) $\dfrac{\cos x\sin x}{\cos y\sin y}$
(C) $\dfrac{2\cos x\sin x}{2\cos y\sin y-1}$
(D) $\dfrac{\sin y\cos y}{1-\cos x\sin x}$
(E) $\dfrac{2\cos y\sin y}{2\cos x\sin x-1}$

---

20. If $f(x)=e^{3x}$, then $f''(\ln 3)=$

(A) 9
(B) 27
(C) 81
(D) 243
(E) 729

GO ON TO THE NEXT PAGE.

21. Find $\frac{dy}{dx}$ if $2y^2 - 6y = x^4 + 2x^3 - 2x - 5$ at (1,1).

(A) –1
(B) –2
(C) –3
(D) –4
(E) –5

---

22. Find $\frac{dy}{dx}$ if $2\sin^3 y + 2\cos^3 x = 2\cos^3 y - 4\sin^3 x$.

(A) $-\frac{\sin 2x(\cos x + 2\sin x)}{\sin 2y(\sin y + \cos y)}$

(B) $\frac{\sin 2x(\cos x + 2\sin x)}{\sin 2y(\sin y + \cos y)}$

(C) $\frac{\sin x \cos x(\cos x + 2\sin x)}{\sin y \cos y(\sin y + \cos y)}$

(D) $-\frac{\sin 2x}{\sin 2y}$

(E) $\frac{\cos 2x(\cos x + 2\sin x)}{\cos 2y(\sin y + \cos y)}$

GO ON TO THE NEXT PAGE.

23. $\int \frac{\ln^3 x}{x} dx =$

(A) $\frac{\ln^3 x}{3} + C$

(B) $\frac{\ln^4 x}{4} + C$

(C) $\frac{\ln^5 x}{5} + C$

(D) $\ln^3 x + C$

(E) $\ln^4 x + C$

---

24. Find the volume of the region formed by the curve $y = x^2$, the x-axis, and the line $x = 3$ when revolved around the y-axis.

(A) $\frac{3}{2}\pi$

(B) $\frac{9}{2}\pi$

(C) $\frac{27}{2}\pi$

(D) $\frac{81}{2}\pi$

(E) $\frac{243}{2}\pi$

GO ON TO THE NEXT PAGE.

25. $\int_0^4 x^3 dx =$

(A) 16
(B) 32
(C) 48
(D) 56
(E) 64

---

26. Is the function $f(x)=\begin{cases} x^3-3, & x<3 \\ 2x+7, & x\geq 3 \end{cases}$ continuous at $x = 3$? If not, what is the discontinuity?

(A) The function is continuous.
(B) Point
(C) Essential
(D) Jump
(E) Removable

---

27. Where does the curve $y=5-(x-2)^{\frac{2}{3}}$ have a cusp?

(A) (0,5)
(B) (5,2)
(C) (2,5)
(D) (5,0)
(E) There is no cusp.

GO ON TO THE NEXT PAGE.

28. $\int (x^2 + 2x)\cos(x^3 + 3x^2)\,dx =$

(A) $\sin(3x^2 + 6x) + C$

(B) $-\frac{1}{3}\sin(x^3 + 3x^2) + C$

(C) $-\sin(x^3 + 3x^2) + C$

(D) $\sin(x^3 + 3x^2) + C$

(E) $\frac{1}{3}\sin(x^3 + 3x^2) + C$

END OF PART A, SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS AB

SECTION I, Part B

Time—50 Minutes

Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

1. The **exact** numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
2. Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

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29. An open top cylinder has a volume of $125\pi \text{ in}^3$. Find the radius required to minimize the amount of material to make the cylinder.

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

GO ON TO THE NEXT PAGE.

30. If the position of a particle is given by $x(t) = 2t^3 - 5t^2 + 4t + 6$, where $t > 0$. What is the distance traveled by the particle from $t = 0$ to $t = 3$?

(A) $\frac{1}{27}$

(B) $\frac{28}{27}$

(C) 20

(D) 21

(E) $\frac{569}{27}$

---

31. At what times, t, are the x- and y-components of the particle's velocity equal if the curve is represented by $x = 2t^3 + 3t^2 - 5$ and $y = t^4 - 4t^3 + 7t^2$?

(A) $t = 0$

(B) $t = \frac{1}{2}$

(C) $t = 4$

(D) $t = 0$ and $t = \frac{1}{2}$

(E) $t = 0,\ t = \frac{1}{2},$ and $t = 4$

GO ON TO THE NEXT PAGE.

32. Find the equation of the line tangent to the graph of $y = 2x - 3x^{-\frac{2}{3}} + 5$ at $x = 8$.

(A) $y = \frac{33}{16}x + \frac{15}{4}$

(B) $y = \frac{15}{4}x + \frac{33}{16}$

(C) $y = \frac{16}{33}x + \frac{4}{15}$

(D) $y = \frac{16}{33}x + \frac{15}{4}$

(E) $y = \frac{33}{16}x + \frac{4}{15}$

---

33. Which point on the curve $y = 5x^3 - 12x^2 - 12x + 64$ has a tangent that is parallel to $y = 3$?

(A) $(0, -2)$

(B) $(2, 32)$

(C) $\left(\frac{2}{5}, 12\right)$

(D) $\left(-2, \frac{288}{25}\right)$

(E) $\left(\frac{2}{5}, \frac{256}{25}\right)$

GO ON TO THE NEXT PAGE.

34. A 50 foot ladder is leaning against a building and being pulled to the ground, so the top is sliding down the building. If the rate the bottom of the ladder is being pulled across the ground is 12 ft/sec, what is the rate of the top of the ladder sliding down the building when the top is 30 ft from the ground?

(A) 12 ft/sec
(B) 9 ft/sec
(C) 20 ft/sec
(D) 9.6 ft/sec
(E) 16 ft/sec

---

35. What is the distance traveled from $t = 0$ to $t = 4$ given the position function, $x(t) = 2t^3 - 9t^2 + 12t + 13$?

(A) 30 units
(B) 32 units
(C) 33 units
(D) 34 units
(E) 35 units

---

36. The tangent to a curve described by $x = 3t^3 - 5t + 2$ and $y = 7t^2 - 16$ is what at $t = 1$?

(A) $-7x + 2y = -18$
(B) $2x - 7y = 18$
(C) $7x + 2y = 18$
(D) $2x + 7y = -18$
(E) $7x - 2y = -18$

GO ON TO THE NEXT PAGE.

37. Approximate $\sqrt{16.04}$.

(A) 4.005
(B) 4.04
(C) 4.02
(D) 4.002
(E) 4.05

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38. Approximate the area under the curve $y = x^2 + 2$ from $x = 1$ to $x = 2$ using four midpoint rectangles.

(A) 4.333
(B) 3.969
(C) 4.719
(D) 4.344
(E) 4.328

---

39. Find the area under the curve $y = x^2 + 2$ from $x = 1$ to $x = 2$.

(A) 4.333
(B) 3.969
(C) 4.719
(D) 4.344
(E) 4.328

GO ON TO THE NEXT PAGE.

40. $\int \frac{3x-2}{(x+2)^2}\,dx =$

(A) $\ln|x+2| + \frac{1}{x+2} + C$

(B) $3\ln|x+2| + \frac{4}{x+2} + C$

(C) $3\ln|x+2| - \frac{4}{x+2} + C$

(D) $-3\ln|x+2| - \frac{4}{x+2} + C$

(E) $-3\ln|x+2| + \frac{4}{x+2} + C$

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41. The side of a cube is increasing at a rate of 3 inches per second. At the instant when the side of the cube is 6 inches long. What is the rate of change (in inches/second) of the surface area of the cube?

(A) 108
(B) 216
(C) 324
(D) 648
(E) 1296

---

42. If the position of a particle is given by $x(t) = 3t^3 - 2t^2 - 16$ where $t > 0$. When does the particle change direction?

(A) $\frac{2}{3}$

(B) $\frac{4}{3}$

(C) $\frac{9}{4}$

(D) 2

(E) 3

GO ON TO THE NEXT PAGE.

43. The radius of a sphere is increased from 9 cm to 9.05 cm. Estimate the change in volume.

(A) $1.25\times10^{-4}\,\text{cm}^3$
(B) $11.3097\,\text{cm}^3$
(C) $16.965\,\text{cm}^3$
(D) $50.894\,\text{cm}^3$
(E) $152.681\,\text{cm}^2$

---

44. Find an equation of the line tangent to the curve represented by $x = 4\cos t + 2$ and $y = 2\sin t$ at $t = \frac{\pi}{3}$.

(A) $y = \frac{\sqrt{3}}{6}x + \frac{5\sqrt{3}}{3}$
(B) $y = -\frac{\sqrt{3}}{6}x + \frac{2\sqrt{3}}{3}$
(C) $y = \frac{\sqrt{3}}{6}x + \sqrt{3}$
(D) $y = -\frac{\sqrt{3}}{6}x + \frac{5\sqrt{3}}{3}$
(E) $y = -\frac{\sqrt{3}}{6}x - \frac{\sqrt{3}}{3}$

---

45. Use differentials to approximate $\sqrt{4.002}$.

(A) 2
(B) 2.0005
(C) 2.005
(D) 2.05
(E) 2.5

STOP

END OF PART B, SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART B ONLY.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

SECTION II
GENERAL INSTRUCTIONS

You may wish to look over the problems before starting to work on them, since it is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

- You should write all work for each part of each problem in the space provided for that part in the booklet. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. You will be graded on the correctness and completeness of your methods as well as your answers. Correct answers without supporting work may not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons and that you clearly identify functions, graphs, tables, or other objects you use.
- You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt (X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

GO ON TO THE NEXT PAGE.

SECTION II, PART A
Time—30 minutes
Number of problems—2

A graphing calculator is required for some problems or parts of problems.

During the timed portion for Part A, you may work only on the problems in Part A.

On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

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1. Water is dripping from a pipe into a container whose volume increases at a rate of $150\,\text{cm}^3/\text{min}$. The water takes the shape of a cone with both its radius and height changing with time.

   (a) What is the rate of change of the radius of the water at the instant the height is 2 cm and the radius is 5 cm? At this instant the height is changing at a rate of $0.5\,\text{cm/min}$.

   (b) The water begins to be extracted from the container at a rate of $E(t) = 75t^{0.25}$. Water continues to drip from the pipe at the same rate as before. When is the water at its maximum volume? Justify your reasoning.

   (c) By the time water began to be extracted, $3000\,\text{cm}^3$ of water had already leaked from the pipe. Write, but do not evaluate, an expression with an integral that gives the volume of water in the container at the time in part (b).

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2. The temperature in a room increases at a rate of $\frac{dT}{dt} = kT$, where k is a constant.

   (a) Find an equation for T, the temperature (in $^{\circ}$F), in terms of t, the number of hours passed, if the temperature is 65 $^{\circ}$F initially and 70 $^{\circ}$F after one hour.
   (b) How many hours will it take for the temperature to reach 85 $^{\circ}$F?
   (c) After the temperature reaches 85 $^{\circ}$F, a fan is turned on and cools the room at a consistent rate of 7 $^{\circ}$F/hour. How long will it take for the room to reach 0 $^{\circ}$F?

GO ON TO THE NEXT PAGE.

SECTION II, PART B
Time—1 hour
Number of problems—4

No calculator is allowed for these problems.

During the timed portion for Part B, you may continue to work on the problems in Part A without the use of any calculator.

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3. Let R be the region enclosed by the graphs of $y = \dfrac{2}{x+1}, y = x^2,$ and the lines $x = 0$ and $x = 1$.

   (a) Find the area of R.
   (b) Find the volume of the solid generated when R is revolved about the x-axis.
   (c) Set up, but do not evaluate, the expression for the volume of the solid generated when R is revolved around the line $x = 2$.

---

4. Consider the equation $x^3 + 2x^2y + 4y^2 = 12$.

   (a) Write an equation for the slope of the curve at any point (x, y).

   (b) Find the equation of the tangent line to the curve at $x = 0$.

   (c) If the equation given for the curve is the path a car travels in feet over t seconds, find $\dfrac{d^2y}{dx^2}$ at $\left(0, \sqrt{3}\right)$ and explain what it represents with proper units.

GO ON TO THE NEXT PAGE.

5. Water is filling at a rate of $64\pi \text{ in}^3$ into a conical tank that has a diameter of 36 in at its base and whose height is 60 in.

(a) Find an expression for the volume of water (in in^3) in the tank in terms of its radius.
(b) At what rate is the radius of the water expanding when the radius is 20 in.
(c) How fast in (in/sec) is the height of the water increasing in the tank when the radius is 20 in?

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6. If a ball is accelerating at a rate given by $a(t) = -64\frac{\text{ft}}{\text{sec}^2}$, the velocity of the ball is $96\frac{\text{ft}}{\text{sec}}$ at time $t = 1$, and the height of the ball is 100 ft at $t = 0$, what is

(a) The equation of the ball's velocity at time t ?
(b) The time when the ball is changing direction?
(c) The equation of the ball's height?
(d) The ball's maximum height?

STOP

END OF EXAM