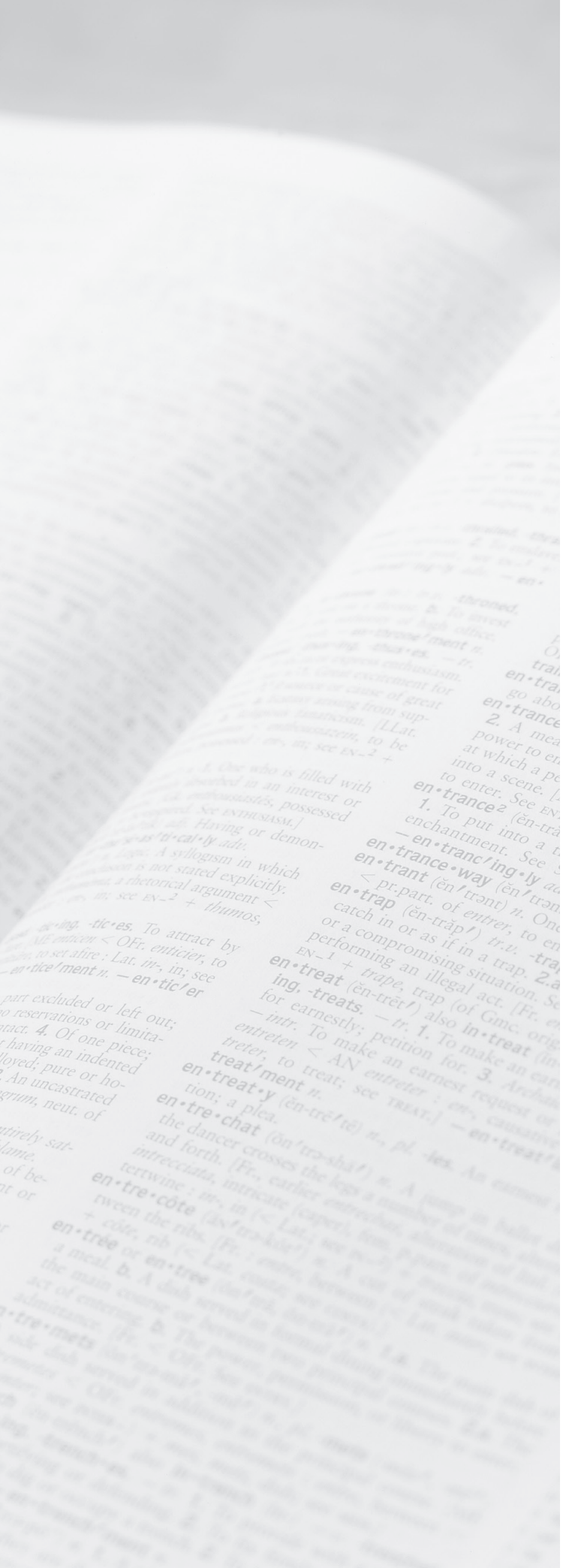

# Chapter 25
# AB & BC Calculus Free Response Drill

# AB & BC CALCULUS FREE RESPONSE DRILL

1. Let $f$ be a function defined by $f(x) = \dfrac{1}{(x^2 - 9)}$.

   (a) What is the domain of $f$ ? Justify your answer.

   (b) Discuss the symmetry of $f$. Justify your answer.

   (c) For what interval(s) is $f$ increasing? Justify your answer.

   (d) For what interval(s) is $f$ concave down?

2. Let $f$ be the function defined by $f(x) = 2 - 15x + 9x^2 - x^3$.

   (a) Find the $x$- and $y$-coordinates of the relative maxima and relative minima. Justify your answer.

   (b) Find an equation for the line normal to $f$ at (2, 1). Justify your answer.

   (c) Find the $x$- and $y$-coordinates of any points of inflection. Justify your answer.

3. The velocity of a particle is given by $v(t) = 3t^2 - 12t + 9$, where $t$ is measured in seconds and $v$ is measured in meters/second.

   (a) Find the position function if $s(3) = 0$.

   (b) When is the particle moving forward?

   (c) Find the total distance traveled by the particle during the first five seconds.

4. Let $f$ be defined by the function $f(x) = 4x - x^2$.

   (a) Find the average value of the function on the interval [0,4].

   (b) Let R be the region bounded by the $x$-axis and the graph of $f(x) = 4x - x^2$. Find the area of the region R.

   (c) Let R be the region bounded by the $x$-axis and the graph of $f(x) = 4x - x^2$. Find the volume of the solid formed by revolving the region R around the $x$-axis.

   (d) Find the $x$- and $y$-coordinates of the absolute maximum of $f$ from [0,4].

5. Let a curve be defined as $x^2 + 4xy + y^2 = 13$.

   (a) Find the equation of the line tangent to the curve at the point (2,1).

   (b) Find the equation of the line normal to the curve at the point (1,2).

   (c) Find $\frac{d^2y}{dx^2}$ at (2,1).

6. Let $y = x^4 + 4x^3$.

   (a) Discuss the domain and symmetry of $y$. Justify your answer.

   (b) Find the $x$- and $y$-intercepts of $y$. Justify your answer.

   (c) Find the intervals on which $y$ is increasing or decreasing and state any relative extrema. Justify your answer.

   (d) Find the intervals of concavity and the point(s) of inflection of $y$. Justify your answer.

7. A particle moves according to the function $f(t) = t^3 - 12t^2 + 36t$, where $t$ is measured in seconds and $f$ in feet.

   (a) Find the velocity and acceleration at time $t$. Justify your answer.

   (b) What is the velocity after 3 seconds? The acceleration after 3 seconds? Is the particle speeding up or slowing down?

   (c) When is the particle at rest?

   (d) Find the total distance traveled by the particle during the first 8 seconds.

8. Consider the equation $f(x) = 2\cos x + \cos^2 x$ on the interval $[0, 2\pi]$.

   (a) Find the intervals where the function is increasing or decreasing. Justify your answer.

   (b) Find the intervals of concavity of $f(x) = 2\cos x + \cos^2 x$. Justify your answer.

   (c) State the $x$- and $y$-coordinates for any relative extrema and point(s) of inflection. Justify your answer.

9. Consider the equation $y = x\sqrt{(x+3)}$.

   (a) What is the domain of $y$? Justify your answer.

   (b) Find the relative extrema of $y$. Justify your answer.

   (c) Find the intervals of concavity and points of inflection, if any. Justify your answer.

10. Water is leaking out of an inverted conical tank at a rate of 10,000 $cm^3$/min at the same time that water is being pumped into the tank at a constant rate. The tank has a height of 6 m and the diameter at the top is 4 m.

    (a) Find an expression for the volume in terms of the height, $h$.

    (b) Find an expression for the volume in terms of the radius, $r$.

    (c) If the water level is rising at a rate of 20 cm/min when the height of the water is 2 m, find the rate at which water is being pumped into the tank.

11. Bacteria grows in a petri dish. The rate of growth of the bacteria is $\frac{dB}{dt} = kB$, where $k$ is a constant.

    (a) Find an expression for $B$, the number of cells in the dish (in thousands), in terms of $t$, the number of minutes passed, if the number of cells is 30 thousand initially and 60 thousand after 1 minute.
    (b) In how many minutes will the number of cells be 300 thousand?

12. If the velocity of the particle traveling along the $x$-axis is given by $v(t) = \frac{2t-5}{t^2+10t+24}$:

    (a) Find the distance the particle travels from $t = 0$ to $t = 5$.
    (b) What is the formula for the acceleration of the particle? What is the acceleration at $t = 3$?
    (c) At what time(s) is the particle's speed decreasing?

13. Consider the parametric functions $x = 3t^3 - 3t$ and $y = 2t^2 + 6t - 4$ that describes the curve of a particle.

(a) Find $\frac{dx}{dt}$ and $\frac{dy}{dt}$.
(b) What is the slope of the curve at $t = 3$?
(c) Find the equation of the normal line to the curve at $t = 3$.
(d) Find $\frac{d^2y}{dx^2}$ at $t = 3$.

14. Let $F(x) = \int_0^x \left[\sin 2t + t^2\right] dt$ on the closed interval $[0, 2\pi]$.

(a) Approximate $F(\pi)$ using six inscribed trapezoids.
(b) Find $F'(2\pi)$.
(c) Find the average value of $F'(x)$ on the interval $[0, 2\pi]$.

15. If the acceleration of a train is given by $a(t) = 24t$ m/sec $^2$. The velocity of the train is 60 m/sec at $t = 0$. If the train has traveled 72 m after 2 sec, find:

(a) The equation for the train's velocity at time $t$.
(b) The speed of the train at $t = 10$.
(c) The distance the train travels from $t = 0$ to $t = 10$.

16.

| $x$ | $f(x)$ | $f'(x)$ | $f''(x)$ | $f'''(x)$ | $f^{(4)}(x)$ |
|---|---|---|---|---|---|
| –1 | 3 | –2 | –16 | 72 | –168 |
| 0 | 0 | 0 | 12 | 24 | 72 |
| 1 | 15 | 46 | 112 | 216 | 312 |

Let $f$ be a function that is differentiable on all orders for $x > 0$. Selected values of $f$ and its first four derivatives are given in the table above. The function and first two derivatives are increasing on the interval $-1 \le x \le 1$.

(a) Write the second-degree Taylor polynomial for $f$ about $x = -1$ and use it to approximate $f(-0.8)$. Is the approximation greater than or less than the true value?
(b) Write the fourth-degree Taylor polynomial for $f$ about $x = -1$ and use it to approximate $f(-0.8)$.
(c) Use the Lagrange error bound to show that the fourth-degree Taylor polynomial for $f$ about $x = -1$ approximates $f(-0.8)$ with an error less than –0.01.