

# AB Calculus Diagnostic Test

## AP® Calculus AB Exam

**SECTION I: Multiple-Choice Questions** 

### DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

## At a Glance

#### **Total Time**

1 hour and 45 minutes **Number of Questions** 

**Percent of Total Grade** 50%

**Writing Instrument** 

Pencil required

#### **Instructions**

Section I of this examination contains 45 multiple-choice questions. Fill in only the ovals for numbers 1 through 45 on your answer sheet.

#### CALCULATORS MAY NOT BE USED IN THIS PART OF THE EXAMINATION.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding oval on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

#### Sample Question

#### Sample Answer

Chicago is a







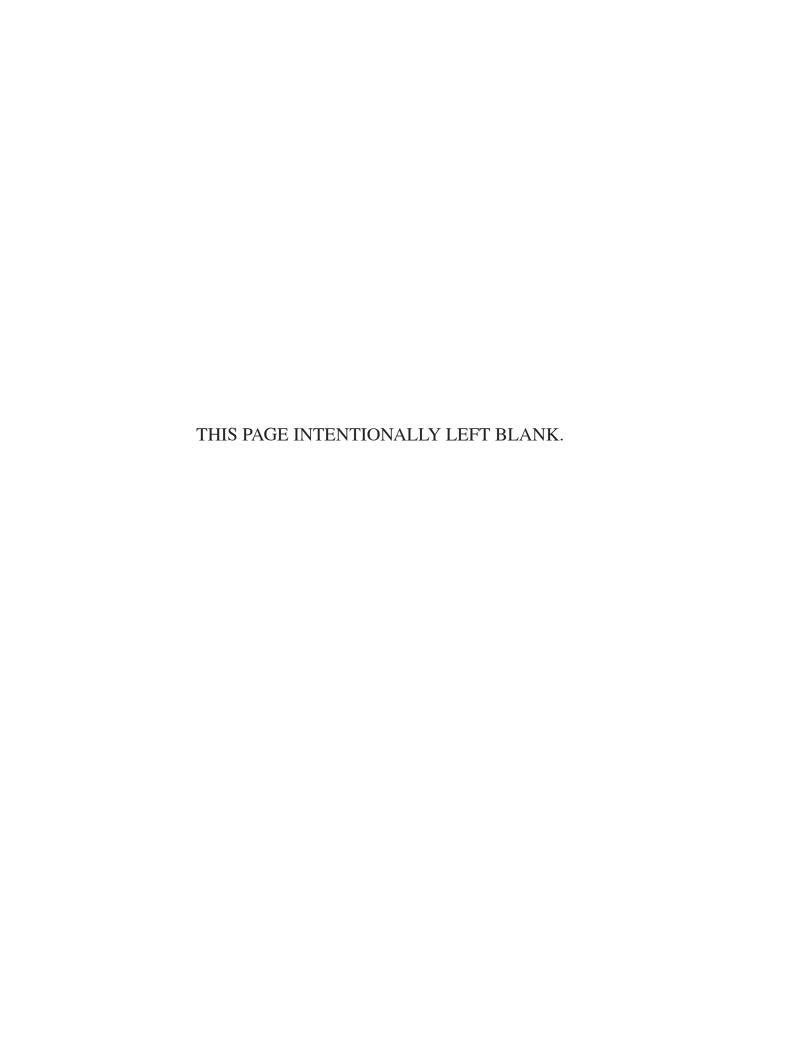


- (A) state
- (B) city
- (C) country
- (D) continent
- (E) village

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all the multiple-choice questions.

## About Guessing

Many candidates wonder whether or not to guess the answers to questions about which they are not certain. Multiple choice scores are based on the number of questions answered correctly. Points are not deducted for incorrect answers, and no points are awarded for unanswered questions. Because points are not deducted for incorrect answers, you are encouraged to answer all multiple-choice questions. On any questions you do not know the answer to, you should eliminate as many choices as you can, and then select the best answer among the remaining choices.



#### **CALCULUS AB**

#### SECTION I, Part A

Time—55 Minutes

Number of questions—28

### A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which f(x) is a real number.

- $1. \quad \lim_{x \to 0} \frac{\sin 2x}{\cos x} =$ 
  - (A) 1
  - (B) 2
  - (C) 0
  - (D) nonexistent
  - (E) 2
- 2.  $\lim_{x \to \infty} \frac{x^4 + x^2 + 1}{3x^4 x^3 + 2x^2 + 5x 5} =$ 

  - (B) 0
  - (C)  $\infty$
  - (D) 3
  - (E) The limit does not exist.

3. At what point does the following function have a removable discontinuity?

$$f(x) = \frac{x^2 + x - 2}{x^2 + 7x + 10}$$

- (A) (-5,-1)
- (B) (-2,-1)
- (C) (-2,1)
- (D) (1,1)
- (E) (1,-1)
- 4. Which of the following functions is NOT continuous at x = -3?

(A) 
$$f(x) = \begin{cases} \frac{x^3 + x - 2}{x^2 - 6x + 9}, & x < -3\\ 3x^3 + 2x, & x \ge -3 \end{cases}$$

(B) 
$$g(x) = \begin{cases} x^2, & x < -3 \\ 9, & x = -3 \\ |3x|, & x > -3 \end{cases}$$

(C) 
$$h(x) = \frac{2x^2 - 8x + 6}{x^2 + x - 2}$$

(D) 
$$j(x) = \sqrt{x-2}$$

(E) 
$$k(x) = (x+3)^2$$

## **Section I**

- 5. Which of the following functions is continuous at x = -3?
  - (A)  $f(x) = \begin{cases} \frac{x^3 + x 2}{x^2 6x + 9}, & x < 0\\ 3x^3 + 2x, & x \ge 0 \end{cases}$
  - (B)  $g(x) = \begin{cases} x^2 + 1, & x < -3 \\ 9, & x = -3 \\ |3x|, & x > -3 \end{cases}$
  - (C)  $h(x) = \frac{x^2 + x 2}{2x^2 + 4x 6}$
  - (D)  $j(x) = (x-2)^{\frac{3}{2}}$
  - (E)  $k(x) = (\sqrt[3]{x+2})^2$
- 6. What is  $\lim_{h \to 0} \frac{\sec\left(\frac{\pi}{2} + h\right) \sec\left(\frac{\pi}{2}\right)}{h}$ ?
  - (A)  $\frac{\pi}{2}$
  - (B) 0
  - (C)  $\frac{\sqrt{2}}{2}$
  - (D) 1
  - (E) The limit does not exist.

7. If 
$$f(x) = (2x^3 + 33)(\sqrt[5]{x} - 2x)$$
, then  $f'(x) =$ 

(A) 
$$\left(2x^3 + 33\right)\left(\frac{1}{5\sqrt[5]{x^4}} - 2\right) + 6x^2\left(\sqrt[5]{x} - 2x\right)$$

(B) 
$$\left(2x^3 + 33\right)\left(\frac{1}{5\sqrt[5]{x^4}} - 2\right) + 6x^3\left(\sqrt[5]{x} - 2x\right)$$

(C) 
$$\left(2x^3 + 33\right)\left(\frac{1}{5}\sqrt[5]{x^4} - 2\right) + 6x^2\left(\sqrt[5]{x} - 2x\right)$$

(D) 
$$\left(2x^3 + 33\right)\left(\frac{1}{5\sqrt[5]{x^4}} - 2\right) + 66x^2\left(\sqrt[5]{x} - 2x\right)$$

(E) 
$$\left(2x^3 + 33\right)\left(5\sqrt[5]{x^4} - 2\right) + 6x^2\left(\sqrt[5]{x} - 2x\right)$$

8. If 
$$y = \left(\frac{x^3 - 1}{x^2 + x}\right)^4$$
, then  $\frac{dy}{dx} =$ 

(A) 
$$4\left(\frac{x^3-1}{x^2+x}\right)^5\left(\frac{3x^2(x^2+x)-(x^3-1)(2x+1)}{(x^2+x)^2}\right)$$

(B) 
$$\left(\frac{3x^2(x^2+x)-(x^3-1)(2x+1)}{(x^2+x)^2}\right)$$

(C) 
$$4\left(\frac{3x^2(x^2+x)-(x^3-1)(2x+1)}{(x^2+x)^2}\right)$$

(D) 
$$\left(\frac{x^3-1}{x^2+x}\right)^3 \left(\frac{3x^2(x^2+x)-(x^3-1)(2x+1)}{(x^2+x)^2}\right)$$

(E) 
$$4\left(\frac{x^3-1}{x^2+x}\right)^3\left(\frac{3x^2(x^2+x)-(x^3-1)(2x+1)}{(x^2+x)^2}\right)$$

## **Section I**

- 9. Find the second derivative of  $x^2y^2 = 2$  at (2,1).
  - (A)
  - (B) -2

  - (D)
  - (E)  $-\frac{1}{2}$
- 10. If the line  $y = ax^2 + bx + c$  goes through the point (2,1) and is normal to  $y = \frac{1}{3}x + 2$  at the point (0,2), then a = ?

  - (B)
  - (C)
  - (D)  $-\frac{4}{5}$
  - (E)
- 11. If  $\frac{d}{dx} f(x) = 2g(x)$  and if  $h(x) = x^3$ , then  $\frac{d}{dx} f(h(x)) =$ 
  - (A)  $6x^2g(x^3)$
  - (B)  $2g(x^3)$
  - (C)  $2x^2g(x^3)$
  - (D)  $6g(x^3)$
  - (E)  $2x^3g(x^3)$

- 12. Which of the following statements about the function given by  $f(x) = \frac{6}{5}x^5 2x^3$  is true?
  - (A) The function has no relative extrema.
  - (B) The graph has one point of inflection and two relative extrema.
  - (C) The graph has three points of inflection and one relative extremum.
  - (D) The graph has three points of inflection and two relative extrema.
  - (E) The graph has two points of inflection and two relative extrema.
- 13.  $\int (2x-5)^3 dx =$

(A) 
$$\frac{(2x-5)^4}{8} + C$$

(B) 
$$\frac{(2x-5)^4}{4} + C$$

(C) 
$$\frac{(2x-5)^4 x^2}{4} + C$$

(D) 
$$\frac{(2x-5)^4}{2} + C$$

(E) 
$$\frac{(2x-5)^4}{6} + C$$

14.  $\int 4x^2 \left(\frac{4}{3}x^3 - 6\right)^9 dx =$ 

(A) 
$$4x^{20} + C$$

(B) 
$$\frac{\left(\frac{4}{3}x^3 + 6\right)^{10}}{10} + C$$

(C) 
$$9\left(\frac{4}{3}x^3+6\right)^8+C$$

(D) 
$$10\left(\frac{4}{3}x^3+6\right)^{10}+C$$

(E) 
$$\left(\frac{4}{3}x^3+6\right)^{10}+C$$

- 15. Find the average value of  $f(x) = 3x^2 \sin x^3$  on the interval  $\left[0, \sqrt[3]{\frac{\pi}{2}}\right]$ .

  - (B)  $2\sqrt[3]{\frac{\pi}{2}}$

  - (D)  $\frac{\pi}{2}$
  - (E) 1

- 16. Find  $\frac{d}{dx} \int_0^{3x^2} t^2 + 4t \, dt$ .
  - (A)  $9x^4 + 12x^2$
  - (B)  $6x(9x^3 + 12x^2)$
  - (C)  $6x^2(9x^4 + 12x^2)$
  - (D)  $90x^3$
  - (E)  $54x^5 + 72x^3$

$$17. \quad \int \frac{\sec x}{\csc x} dx =$$

- (A)  $-\ln|\cos x| + C$
- (B)  $\ln \left| \sin x \right| + C$
- (C)  $\ln \left| \sec x + \tan x \right| + C$
- (D)  $-\ln|\csc x + \cot x| + C$
- (E)  $-\ln|\sin x| + C$
- 18. What is the area between  $y = x^3$  and y = x?
  - (A) 0
  - (B) 1

  - (C)  $\frac{1}{4}$ (D)  $\frac{3}{4}$ (E)  $\frac{1}{2}$
- 19. Find the volume of the region bounded by  $y = (x 5)^3$ , the x-axis, and the line x = 10 as it is revolved around the line x = 2. Set up, but do not evaluate the integral.

(A) 
$$2\pi \int_{5}^{10} x(x-5)^3 dx$$

(B) 
$$2\pi \int_{5}^{10} (x-2)(x-5)^3 dx$$

(C) 
$$2\pi \int_{2}^{10} x(x-5)^3 dx$$

(D) 
$$2\pi \int_{2}^{10} (x-2)(x-5)^{3} dx$$

(E) 
$$2\pi \int_0^{10} (x-2)(x-5)^3 dx$$

## **Section I**

- $20. \quad \int 3x^2(x^3 3)^7 dx =$ 
  - (A)  $8(x^3 3)^8 + C$
  - (B)  $\frac{(x^3-3)^8}{8} + C$
  - (C)  $(x^3 3)^8 + C$
  - (D)  $x^3 + C$
  - (E)  $x^3(x^3-3)^8 + C$
- 21. Find the equation for the normal line to  $y = 3x^2 6x$  at (2,0).
  - (A)  $y = -6x \frac{1}{3}$
  - (B)  $y = -\frac{1}{6}x + \frac{1}{3}$
  - (C)  $y = \frac{1}{6}x + \frac{1}{3}$
  - (D) y = 6x + 3
  - (E)  $y = -\frac{1}{6}x \frac{1}{3}$
- $22. \quad \lim_{x \to \infty} \frac{\sin x \cos x + \sin x}{x^2} =$ 
  - (A) 0
  - (B) -1
  - (C)  $-\infty$
  - (D)  $\infty$
  - (E)

- 23.  $\frac{d}{dx} \int_3^{x^3} (2-t^2) dt =$ 
  - (A)  $6x^2 + 3x^8$
  - (B)  $6x^2 9x^4$
  - (C)  $2 x^2$
  - (D)  $6x^2 3x^8$
  - (E)  $2 x^6$

- $\int \frac{dx}{\sqrt{9-x^2}} =$ 24.
  - (A)  $\frac{1}{9}\sin^{-1}\left(\frac{x}{3}\right) + C$
  - (B)  $\sin^{-1}\left(\frac{x}{3}\right) + C$
  - (C)  $\frac{1}{3}\sin^{-1}\left(\frac{x}{3}\right) + C$
  - (D)  $\tan^{-1}\left(\frac{x}{3}\right) + C$
  - (E)  $\frac{1}{9} \tan^{-1} \left( \frac{x}{3} \right) + C$
- 25. Find f'(x) for  $f(x) = x^3 + 2x$  when x = 1.
  - (A) 2
  - (B) 3
  - (C) 4
  - (D) 5 (E) 6

$$26. \quad \frac{d^2}{dx^2} \left( x^3 + 2x^2 - 14 \right) =$$

- (A) 6*x*
- (B) 6x + 4
- (C)  $3x^2 + 4x$
- (D)  $3x^2$
- (E) 4x
- 27. Find the absolute maximum on the interval [-2, 2] for the curve  $y = 4x^5 10x^2 8$ .
  - (A) -2
  - (B) -1
  - (C) 0
  - (D) 1
  - (E) 2
- 28. Find  $\frac{d^2y}{dx^2}$  at (-1,-2) if  $3x^3 2x^2 + x = y^3 + 2y^2 + 3y$ .
  - (A)  $\frac{10}{7}$
  - (B) 2
  - (C)  $-\frac{10}{7}$
  - (D) -2
  - (E) 0

**END OF PART A, SECTION I** IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

#### CALCULUS AB

#### SECTION I, Part B

Time—50 Minutes

Number of questions—17

## A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

#### In this test:

- The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which f(x) is a real number.

29. If 
$$f(x) = x^{-3} + 3\sqrt{x} + 5\pi - e^2$$
, then  $f'(x) =$ 

(A) 
$$\frac{3}{2\sqrt{x}} - 3x^4$$

(B) 
$$\frac{3}{2}x^{\frac{1}{2}} - 3x^{-4}$$

(C) 
$$3x^{-4} + \frac{3}{2\sqrt{x}}$$

(D) 
$$-\frac{3}{x^4} + \frac{3}{2\sqrt{x}}$$

(E) 
$$3x^2 + \frac{3}{2\sqrt{x}}$$

- 30. Find the value of c that satisfies Rolle's theorem for  $f(x) = 2x^4 16x$  on the interval [0,2].
  - (A) 2
  - (B)  $\left(-2\right)^{-\frac{1}{3}}$
  - (C)  $2^{-\frac{1}{3}}$
  - (D)  $(-2)^{\frac{1}{3}}$
- 31. Find the absolute maximum of  $y = \frac{5}{3}x^3 x^2 7x$  on the interval [-2,2].
  - (A)
  - (B)
  - (C)
  - (D)
  - (E)
- 32. Approximate the area under the curve  $y = x^2 + 2$  from x = 1 to x = 2 using four right-endpoint rectangles.
  - (A) 4.333
  - (B) 3.969
  - (C) 4.719
  - (D) 4.344
  - (E) 4.328

- 33. Approximate the area under the curve  $y = x^2 + 2$  from x = 1 to x = 2 using four inscribed trapezoids.
  - (A) 4.333
  - (B) 3.969
  - (C) 4.719
  - (D) 4.344
  - (E) 4.328
- 34. Evaluate  $\int_{-\pi}^{\pi} \frac{x \cos x^2}{4} dx.$ 
  - (A) 1
  - (B) 2
  - (C) -1
  - (D)  $\frac{1}{8}$
  - (E) 0
- 35. Suppose  $F(x) = \int_0^x t^3 + t \, dt$ . What is the change in F(x) as t increases from 1 to 4.
  - (A) 72
  - (B) 71.25
  - (C) 24.75
  - (D) 6
  - (E) 0.75

- 36. In the xy-plane, 2x + y = k is tangent to the graph of  $y = 2x^2 8x + 14$ . What is the value of k?

  - (A)  $\frac{3}{2}$  (B)  $\frac{13}{2}$
  - (C) 5

37. The function f is continuous on the closed interval [0,4] and twice differentiable over the open interval, (0, 4). If f'(2) = -5and f''(x) > 0 over the interval (0, 4), which of the following could be a table of values for f?

(A)

х	f(x)
0	10
1	7.5
2	6
3	4.5
4	2

(B)

Х	f(x)
0	10
1	7.5
2	6.5
3	3.5
4	2

(C)

х	f(x)
0	10
1	7
2	4.5
3	3
4	2.5

(D)

х	f(x)
0	10
1	8
2	6
3	4
4	2

(E)

х	f(x)
0	10
1	8.5
2	5.5
3	3.5
4	2.5

- 38. When is the particle whose path is described by  $x(t) = 2t^3 \frac{21}{2}t^2 + 9t 16$ , from t > 0, slowing down?
  - (A) 0 < t < 3

  - (B)  $\frac{7}{4} < t < 3$ (C)  $\frac{1}{2} < t < \frac{7}{4}$
  - (D) t > 3
  - (E)  $\frac{1}{2} < t < 3$
- 39. What is the are enclosed by the curve  $f(x) = 4x^2 x^4$  and the x-axis.
  - (A)  $\frac{128}{15}$

  - (C)  $\frac{64}{5}$

  - (E) 0
- 40. Which of the following is an asymptote for the curve  $y = \frac{3x}{x+7}$ ?
  - (A) x = 7
  - (B) y = 3
  - (C) y = -7
  - (D) x = 3
  - (E) x = -3

- $41. \quad \int \frac{x}{x^2 7} dx =$ 
  - (A)  $\ln \left| \frac{x}{x^2 7} \right| + C$
  - (B)  $2 \ln |x^2 7| + C$
  - (C)  $\ln \left| x^2 7 \right| + C$
  - (D)  $\frac{1}{2} \ln |x^2 7| + C$
  - (E)  $\ln |x| + C$
- 42. What is the area between the curves  $y = x^3 2x^2 5x + 6$  and  $y = x^2 x 6$  from x = -2 to x = 3?
  - (A) 30
  - 367 (B)
  - (C) 32
  - (D)
  - (E) 34
- 43. Find the average value of  $f(x) = (3x-1)^3$  on the interval from x = -1 to x = 3.
  - (A) 10.667
  - (B) 12
  - (C) 9.333
  - (D) 15
  - (E) 18

- 44. The curve  $y = ax^2 + bx + c$  passes through the point (1,5) and is normal to the line -x + 5y = 15 at (0,3). What is the equation of the curve?
  - (A)  $y = 7x^2 0.2x + 3$
  - (B)  $y = 2.2x^2 0.2x + 3$
  - (C)  $y = 7x^2 + 5x + 3$
  - (D)  $y = 7x^2 5x + 3$
  - (E)  $y = 5x^2 7x + 3$
- 45.  $\lim_{x \to 1} ((x^3 + 2x^2 3)(x^{-2} + 7x)) =$ 
  - (A) -1
  - (B) 0
  - (C) 1
  - (D) 8
  - (E) ∞

# **STOP**

**END OF PART B, SECTION I** IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART B ONLY. DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

## SECTION II **GENERAL INSTRUCTIONS**

You may wish to look over the problems before starting to work on them, since it is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

## A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

- You should write all work for each part of each problem in the space provided for that part in the booklet. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. You will be graded on the correctness and completeness of your methods as well as your answers. Correct answers without supporting work may not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons and that you clearly identify functions, graphs, tables, or other objects you use.
- You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example,  $\int_{0}^{3} x^{2} dx$  may not be written as fnInt (X<sup>2</sup>, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which f(x) is a real number.

## SECTION II, PART A Time—30 minutes

Number of problems—2

A graphing calculator is required for some problems or parts of problems.

During the timed portion for Part A, you may work only on the problems in Part A.

On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

- 1. A cylindrical drum is filling with water at a rate of  $25\pi$  in<sup>3</sup>/sec.
  - (a) If the radius of the cylinder is 1/3 the height, write an expression for the volume of water in terms of the height at any
  - (b) At what rate is the height changing when the height is 10 in?
  - (c) What is the height of the water when it is increasing at a rate of 12 in/sec?
- 2. The function f is defined by  $f(x) = (9 x^2)^{\frac{3}{2}}$  for  $-3 \le x \le 3$ .
  - (a) Find f'(x).
  - (b) Write an equation for the line tangent to the graph of f at x = -2.
  - (c) Let *g* be the function defined by  $g(x) = \begin{cases} f(x), & \text{for } -3 \le x \le -2 \\ 2x + 9, & \text{for } -2 < x \le 3 \end{cases}$ . Is *g* continuous at x = -2? Use the definition of continuous ity to explain your answer.
  - (d) Find the value of  $\int_0^3 3x (9-x^2)^{\frac{3}{2}} dx$ .

## SECTION II, PART B Time—1 hour

# Number of problems—4

## No calculator is allowed for these problems.

During the timed portion for Part B, you may continue to work on the problems in Part A without the use of any calculator.

- 3. A particle moves with velocity  $v(t) = 9t^2 + 18t 7$  for  $t \ge 0$  from an initial position of s(0) = 3.
  - (a) Write an equation for the position of the particle.
  - (b) When is the particle changing direction?
  - (c) What is the total distance covered from t = 2 to t = 5?
- 4. Let f be the function given by  $f(x) = -2x^4 + 6x^2 + 2$ .
  - (a) Find the equation for the line normal to the graph at (1,6).
  - (b) Find the x and y coordinates of the relative maximum and minimum points.
  - (c) Find the x and y coordinates of the points of inflection.
- 5. Consider the curve given by  $x^3y^2 5x + y = 3$ .
  - (a) Find  $\frac{dy}{dx}$ .
  - (b) Find  $\frac{d^2y}{dx^2}$ .
  - (c) Find the equation of the normal lines at each of the two points on the curve whose x-coordinate is -1.

## Section II

- 6. Let f be the function given by  $f(x) = 3x^3 6x^2 + 4x$ .
  - (a) Find an equation for the normal line at x = 2.
  - (b) Where are the relative maxima and minima of the curve, if any exist? Verify your answer.
  - (c) Where are the points of inflection? Verify your answer. If there are none, explain why.

**STOP** 

**END OF EXAM**